Lesson 5

A-Level Applied Mathematics : Mechanics : Year 2
Variable Acceleration (Kinematics IV)

5.1 Integration and Variable Acceleration

Example
A particle's velocity, $\boldsymbol{v}$, is given by;

$$\boldsymbol{v} = \left(12t^2\,\mathbf{i} + 5\mathbf{j} \right) \text{ m s}^{-1}$$

(i) By integration, find an expression for the position of the particle, $\boldsymbol{r}$.
Call the unknown constants of the integration c_x and c_y

(ii) Use the extra information that when $t = 0$, $\boldsymbol{r} = (3\mathbf{i} - 4\mathbf{j})$ metres to determine c_x and c_y and so rewrite your part (i) answer appropriately.

(iii) Find the position of the particle when $t = 3$ seconds.
Give your answer in the form $\boldsymbol{r}_{t=3} = \left(p\mathbf{i} + q\mathbf{j} \right)$ metres.

5.2 Exercise

Question 1
When $t = 0$, a particle has velocity $\boldsymbol{v}_{t=0} = \left(5\mathbf{i} - 6\mathbf{j} \right)$ m s^{-1}
The particle has an acceleration, $\boldsymbol{a}$, given by;

$$\boldsymbol{a} = \left(2t\,\mathbf{i} - 3\mathbf{j} \right) \text{ m s}^{-2}$$

Find the speed of the particle when $t = 4$ seconds.

Question 2

When $t = 0$ particle P is at the position $3\mathbf{i} + 4\mathbf{j}$ relative to fixed point O.
The velocity, $\boldsymbol{v}_P$, at time, t, is given by;

$$\boldsymbol{v}_P = 6t^2\,\mathbf{i} + (2t + 3)\,\mathbf{j}$$

(i) Find the position vector of P relative to O at time t seconds.

When $t = 0$ particle Q is at the position $2\mathbf{i} + 10\mathbf{j}$ relative to O.
The velocity, $\boldsymbol{v}_Q$, at time, t, is given by;

$$\boldsymbol{v}_Q = 3\mathbf{i} - 2\mathbf{j}$$

(ii) Find the position vector of Q relative to O at time t seconds.

(iii) Find the time at which P and Q will collide and also the position vector of their point of collision.

Question 3

When $t = 0$ a particle P is at rest at point A whose position relative to a fixed point O is given by $\boldsymbol{r} = (3\mathbf{i} + 4\mathbf{j})$ metres.

P moves with acceleration, $\boldsymbol{a}$, given by;

$$\boldsymbol{a} = (5t\,\mathbf{i})\ \text{m s}^{-2}$$

Find the distance P moves between $t = 1$ and $t = 2$ seconds.

Question 4

M2 Examination question from January 2012, Q2

A particle P is moving in a plane. At time t seconds, P is moving with velocity $\boldsymbol{v}$ m.s^{-1}, where $\boldsymbol{v} = 2t\,\mathbf{i} - 3t^2\,\mathbf{j}$

Find

(a) the speed of P when $t = 4$

[2 marks]

(b) the acceleration of P when $t = 4$

[3 marks]

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ metres when $t = 1$,

(c) find the position vector of P when $t = 4$

[5 marks]

Question 5

M2 Examination question from May 2009, Q2

A time $t = 0$ a particle P leaves the origin O and moves along the x-axis.
At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2$$

(a) Find the maximum value of v

[4 marks]

(b) Find the time taken for P to return to O.

[5 marks]

Question 6

A-Level Examination Question from October 2021, Paper 32, Q5 (Edexcel)

At time t seconds, a particle P has velocity $\boldsymbol{v}$ m s^{-1}, where

$$\boldsymbol{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$

[2 marks]

(b) Find the value of t when P is moving in the direction of $\mathbf{i} - \mathbf{j}$

[3 marks]

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\boldsymbol{r}$ metres.

When $t = 1$, $\boldsymbol{r} = -\mathbf{j}$

(c) Find an expression for $\boldsymbol{r}$ in terms of t

[3 marks]

(d) Find the exact distance of P from O at the instant when P is moving with speed 10 m s^{-1}

[6 marks]

Question 7

A-Level Examination Question from June 2018, Q6

At time t seconds, where $t \geqslant 0$, a particle P moves in the x-y plane in such a way that its velocity $\mathbf{v}$ m s^{-1} is given by

$$v = t^{-\frac{1}{2}}\,\mathbf{i} - 4t\,\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B.

Find the exact distance AB

[6 marks]