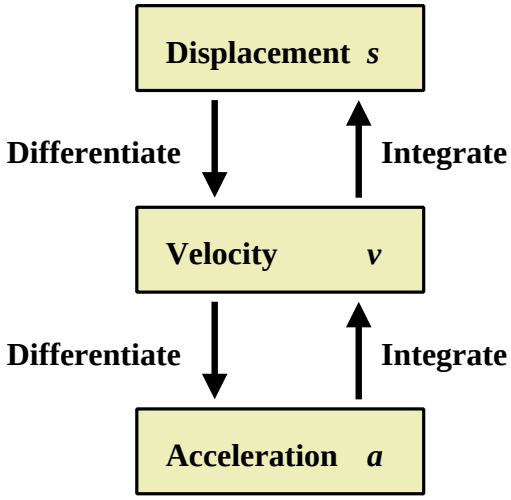
Lesson 6

A-Level Applied Mathematics : Mechanics : Year 2

Variable Acceleration (Kinematics IV)

6.1 Examination Questions on Variable Acceleration

There will be a variable acceleration question in the examination !



When integrating do not forget, for a one dimensional problem, the unknown constant of the integration, c.

In two dimensional problems you need two unknown constants, c_x and c_y

6.2 Example

At time t seconds (where $t \geqslant 0$) the particle P is moving in a plane with acceleration $\boldsymbol{a}$ m s^{-1}, where $\boldsymbol{a} = \left(8t^3 - 6t\right)\mathbf{i} + \left(8t - 3\right)\mathbf{j}$

When $t = 2$, the velocity of P is $(16\mathbf{i} + 3\mathbf{j})$ m s^{-1}

Find the velocity of P after 3 seconds.

[3 marks]

6.3 Exercise

Question 1

A particle P is moving in a plane.

At time t seconds, it's velocity $\boldsymbol{v}$ m s^{-1} is given by,

$$\boldsymbol{v} = 3t\,\mathbf{i} + \frac{1}{2}t^2\,\mathbf{j} \qquad t \geqslant 0$$

When $t = 0$, the position vector of P with respect to a fixed origin is O is $(2\mathbf{i} - 3\mathbf{j})$ metres. Find the position vector of P at time $t = 6$ seconds.

[4 marks]

Question 2

M2 Examination question from June 2006, Q1

A particle P moves on the x-axis.

At time t seconds, its acceleration is $(5 - 2t)$ m.s^{-2} measured in the direction of x increasing. When $t = 0$, its velocity is 6 m.s^{-1} measured in the direction of x increasing.

Find the time when P is instantaneously at rest in the subsequent motion.

[6 marks]

Question 3

M2 Examination question from June 2003, Q1

A particle P moves on the x-axis. At time t seconds the velocity of P is v m.s^{-1} in the direction of x increasing, where $v = 6t - 2t^2$.

When $t = 0$, P is at the origin O. Find the distance of P from O when P comes to instantaneous rest after leaving O.

[5 marks]

Question 4

M2 Examination question from June 2014, Q2

At time t seconds, where $t \geqslant 0$, a particle P is moving on a horizontal plane with acceleration,

$$\left[\left(3t^2 - 4t \right)\mathbf{i} + \left(6t - 5 \right)\mathbf{j} \right] \text{ m s}^{-2}$$

When $t = 3$ the velocity of P is $(11\mathbf{i} + 10\mathbf{j})$ m.s^{-1}

Find

(a) the velocity of P at time t seconds

[5 marks]

(b) the speed of P when it is moving parallel to the vector **i**

[4 marks]

Question 5

M2 Examination question from June 2013, Q3

A particle P moves on the x-axis. At time t seconds the velocity of P is v m.s^{-1} in the direction of x increasing, where

$$v = 2t^2 - 14t + 20, \qquad t \geqslant 0$$

Find

(a) the times when P is instantaneously at rest

[3 marks]

(b) the greatest speed of P in the interval $0 \leqslant t \leqslant 4$

[5 marks]

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 4$

[5 marks]

Question 6

M2 Examination question from June 2015, Q6

A particle P moves on the positive x-axis. The velocity of P at time t seconds is v m.s^{-1}, where

$$v = 2t^2 - 9t + 4$$

When $t = 0$, P is 15 m from the origin O.

Find

(a) the values of t when P is instantaneously at rest

[3 marks]

(b) the acceleration of P when $t = 5$

[3 marks]

(c) the total distance travelled by P in the interval $0 \leqslant t \leqslant 5$

[5 marks]

Question 7

M2 Examination question from June 2004, Q4

At time t seconds, the velocity of a particle P is $\left((4t - 7)\mathbf{i} - 5\mathbf{j} \right)$ m s^{-1}.

When $t = 0$, P is at the point with position vector $\left(3\mathbf{i} + 5\mathbf{j} \right)$ metres relative to a fixed origin, O.

(a) Find an expression for the position vector of P after t seconds, giving your answer in the form $\left(a\,\mathbf{i} + b\,\mathbf{j} \right)$ metres.

[4 marks]

A second particle Q moves with constant velocity $\left(2\mathbf{i} - 3\mathbf{j} \right)$ m s^{-1}.

When $t = 0$, the position vector of Q is $\left(-7\mathbf{i} \right)$ metres

(b) Prove that P and Q collide.

[6 marks]