Lesson 7

A-Level Applied Mathematics : Mechanics : Year 2

Variable Acceleration (Kinematics IV)

7.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 50

Assume that **i** *and* **j** *are horizontal unit vectors due east*
and due North respectively unless the question states otherwise

Question 1

A particle, initially located at $(7\mathbf{i} - 4\mathbf{j})$ metres from a fixed origin *O*, moves with initial velocity $(7\mathbf{i} - 8\mathbf{j})$ m s^{-1}

It is accelerating uniformly at $(-3\mathbf{i} + 5\mathbf{j})$ m s^{-2}

(i) What is the particle's speed 4 seconds later ?

[3 marks]

(ii) On what bearing is the particle moving at that instant ?

[2 marks]

(iii) Also, what then is the particle's location ?

[3 marks]

Question 2

A particle moves with constant acceleration $(3\mathbf{i} - 2\mathbf{j})$ m s^{-2}

At time $t = 0$, it is at the point A and is moving with velocity $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1}

At time $t = T$ seconds, the particle is moving in the direction of vector $(-2\mathbf{i} + 3\mathbf{j})$

(i) Find the value of T

[4 marks]

(ii) Find the velocity of the particle at time T.
Give your answer in the form $p\mathbf{i} + q\mathbf{j}$ where p and q are constants with values that are to be found.

[2 marks]

Question 3

As part of a military naval exercise, a submarine performs a crash dive followed by an emergency surface.

It's vertical depth in metres after t seconds is given by,

$$s = \frac{t^3}{576} - \frac{t^2}{6} \qquad \text{for } 0 \leqslant t \leqslant 96$$

(a) Find the formula for the vertical velocity, v metres per second, t seconds after commencing the exercise.

[2 marks]

(b) Find the formula for the vertical acceleration, a metres per second square, t seconds after commencing the exercise.

[2 marks]

(c) Find the greatest depth, in metres, reached by the submarine during the exercise.

[3 marks]

(d) Find the vertical velocity with which the submarine re-surfaces.
Give this answer in kilometres per hour.

[3 marks]

Question 4

M2 A-level examination question from June 2002, Q2

With respect to a fixed origin O, the velocity $\boldsymbol{v}$ m s^{-1} of a particle P at time t seconds is given by

$$\boldsymbol{v}_P = t^2\,\mathbf{i} + (5 - 4t)\,\mathbf{j}$$

(a) Find the acceleration of P at time t seconds, giving your answer as a vector.

[2 marks]

(b) Find the value of t when P is moving parallel to the vector $\mathbf{i}$

[2 marks]

When $t = 0$, P is at the point with position vector $-3\mathbf{i}$ metres.

(c) Find, to 3 significant figures, the distance OP when $t = 3$

[5 marks]

Question 5

At time t seconds, the velocity of a particle P is $((4t - 3)\mathbf{i} + 4\mathbf{j})$ m s^{-1}.

When $t = 0$, P has position vector $(\mathbf{i} + 2\mathbf{j})$ metres relative to a fixed origin, O.

(a) Find an expression for the position vector of P after t seconds, giving your answer in the form $(a\mathbf{i} + b\mathbf{j})$ metres.

[3 marks]

A second particle Q moves with constant velocity $(5\mathbf{i} + k\mathbf{j})$ m s^{-1}.

When $t = 0$, the position vector of Q is $(11\mathbf{i} + 5\mathbf{j})$ metres

(b) Given that the particles P and Q collide, find:

(i) the value of k

[3 marks]

(ii) the position vector of the point of collision

[2 marks]

Question 6

A particle moves with a velocity $\boldsymbol{v}$ m s^{-1} where t is the time in seconds and,

$$\boldsymbol{v} = \begin{pmatrix} 16t - 9t^2 \\ 7 - t^3 \end{pmatrix}$$

When $t = 0$ the position of the particle relative to a fixed origin is $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ metres.

(i) Find the position of the particle after 3 seconds.

[3 marks]

(ii) Find the magnitude of the acceleration of the particle when $t = 2$ seconds.

[3 marks]

Question 7

A particle P moves so that its acceleration $\boldsymbol{a}$ m s^{-2} at time t seconds, where $t \geqslant 0$, is given by

$$\boldsymbol{a} = 4t\,\mathbf{i} + \frac{5}{\sqrt{t}}\,\mathbf{j}$$

When $t = 0$, the velocity of P is $10\mathbf{i}$ m s^{-1}

Find the speed of P when $t = 5$

[3 marks]