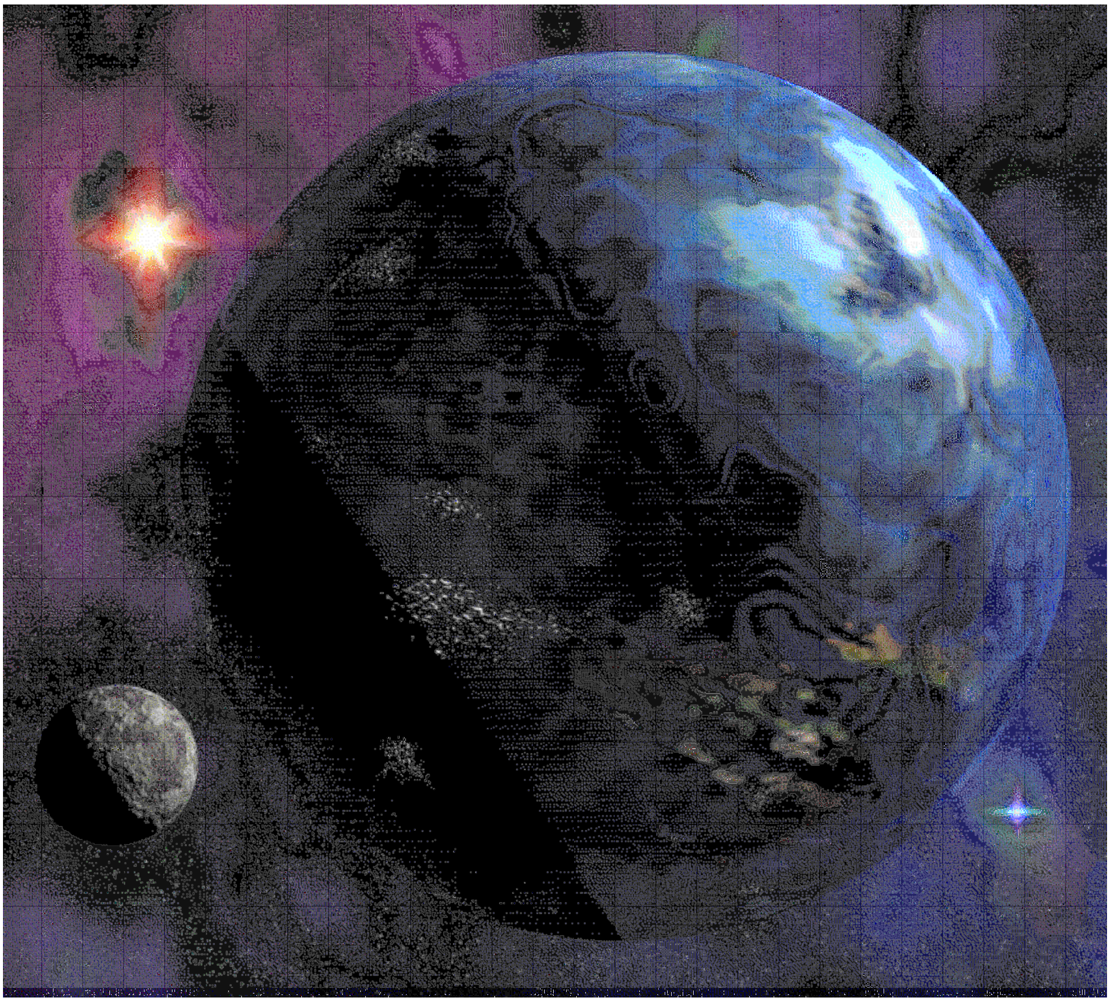
A-Level Applied Mathematics
Mechanics
Year 1

DYNAMICS I



Is it Angels, beating their wings, that keeps the moon moving around the Earth ?

D Y N A M I C S   I

Lesson 1

A-Level Applied Mathematics

Mechanics : Dynamics I : Year 1

1.1 The Puzzle Of The Moon

At the start of the 17th century there were fractious discussions in academic journals about why the Moon kept moving about the Earth. Experience with wheeled vehicles on roads suggested that, over relatively short periods of time, most moving things came to a halt.
In the scientific journals of the time, it was not unusual for a contributor to claim that, for example, it was angels, beating their wings, that kept powering the moon along. Such ideas were considered seriously and debated.

It was part of the genius of Isaac Newton (1642-1727) to realised that it was in the nature of objects to keep moving at constant velocity, and that it required the intervention of a force, such as friction, to slow it down. So, given that the Moon was already in motion, it required nothing to keep it moving; the real surprise would be were it to suddenly stop.

Kinematics is the study of the motion of a particle without regard to any forces.
However, we who have grown up in the “space age” know that

- ◇ to make a stationary object move,
- ◇ to speed up one that is already moving,

or ◇ to slow one down,

a force needs to be applied.

Dynamics is the study of motion when any forces acting are taken into account.

Sir Isaac Newton worked out the insightful way to think about force and motion which is summarised by his famous ***three laws of motion***.

Newton's Three Laws of Motion

1st Law	An object will remain at rest or will continue to move in a straight line at a constant velocity unless it is acted upon by a resultant force.
2nd Law	$F = m\,a$ (An unbalanced force = Mass × Acceleration)
3rd Law	Every action has an equal and opposite reaction.

1.2 The First Law

The first law draws our attention to the concept of “a resultant force”. Given a particle upon which several forces act, the resultant is the single replacement force that would cause the particle to move in the same manner as before the replacement took place. If this replacement single force turns out to be zero then Newton's first law tells us that the object, modelled as a particle, will stay as rest if it is already at rest, or continue to move with constant velocity. Note that moving with constant velocity implies that the acceleration is zero.

1.3 Example

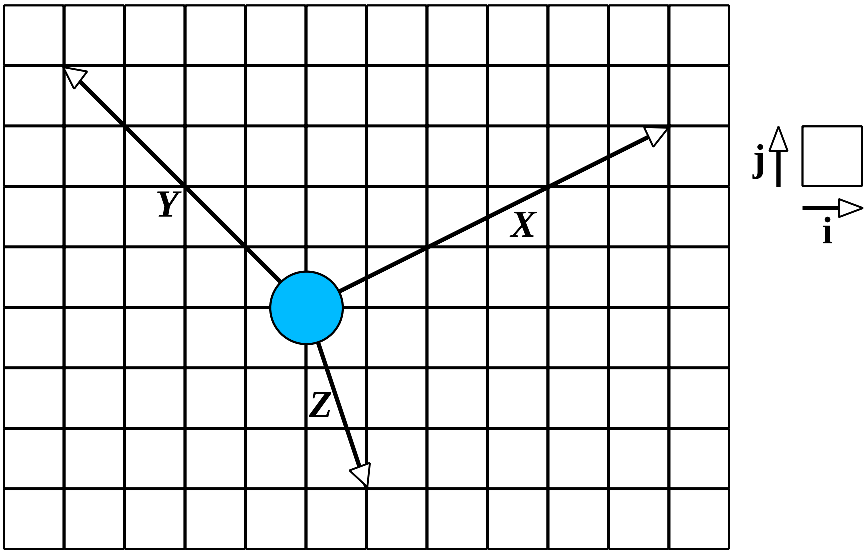
Three forces act upon a particle, floating in space.

The forces, measured in Newtons, are,

$$\boldsymbol{X} = 6\mathbf{i} + 3\mathbf{j} \qquad \boldsymbol{Y} = -4\mathbf{i} + 4\mathbf{j} \qquad \boldsymbol{Z} = \mathbf{i} - 3\mathbf{j}$$

where **i** and **j** are unit vectors in the *x*-axis and *y*-axis directions respectively.

These are draw to scale on the following diagram.



(i) As a vector, what is the resultant force ?

(ii) As a vector, what is the equilibrant ?

(iii) Add in the equilibrant on the diagram, drawn to scale, where the side of each square represents one newton of force.

(iv) What is the magnitude of the equilibrant ?

(v) At what angle to the positive *x*-axis direction does the equilibrant act ?

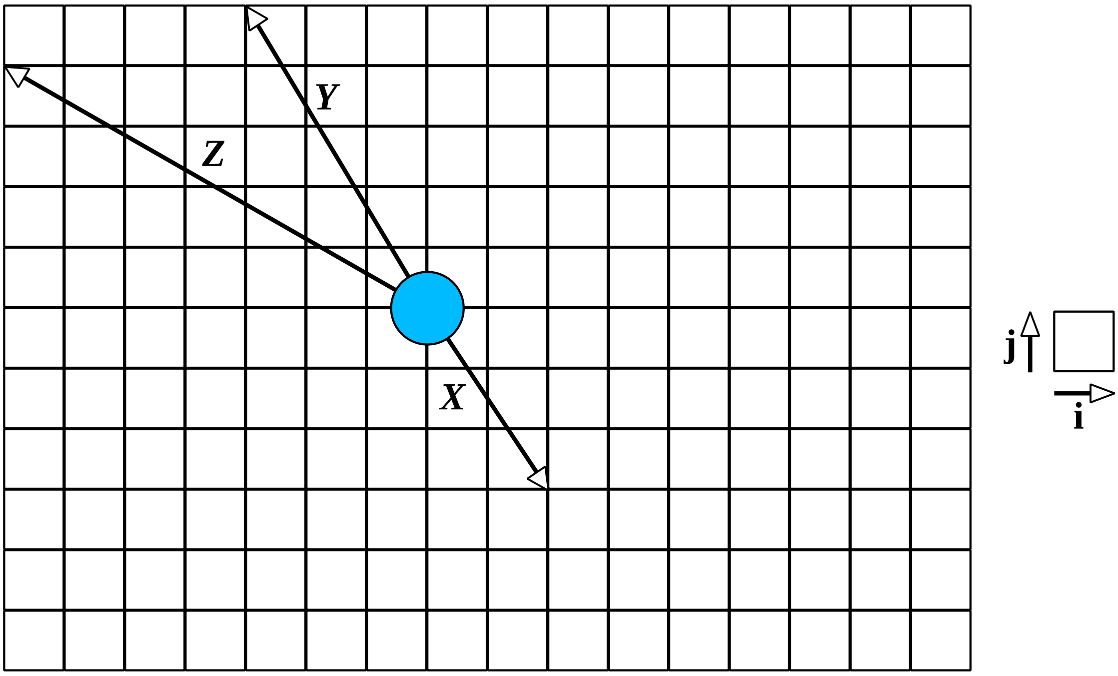
1.4 Exercise

Question 1

Three forces act upon a particle, floating in space (where there is no gravity).
The forces, measured in Newtons, are,

$$\boldsymbol{X} = 2\mathbf{i} - 3\mathbf{j} \qquad \boldsymbol{Y} = -3\mathbf{i} + 5\mathbf{j} \qquad \boldsymbol{Z} = -7\mathbf{i} + 4\mathbf{j}$$

where **i** and **j** are unit vectors in the x-axis and y-axis directions respectively.
These are draw to scale on the following diagram.



(i) As a vector, what is the resultant force ?

(ii) As a vector, what is the equilibrant ?

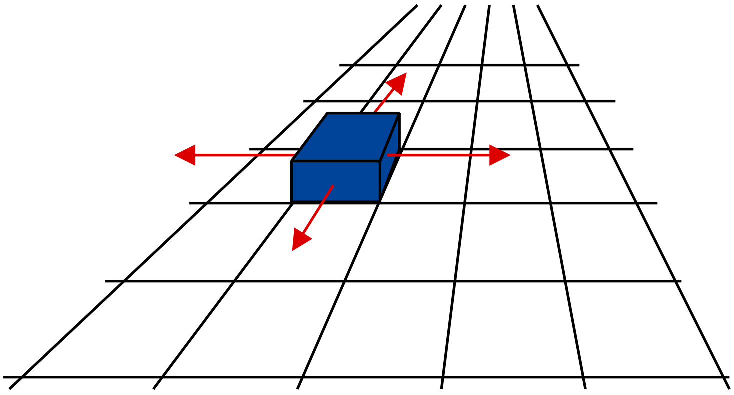
(iii) Add in the equilibrant on the diagram, drawn to scale, where the side of each square represents one newton of force.

(iv) What is the magnitude of the equilibrant ?

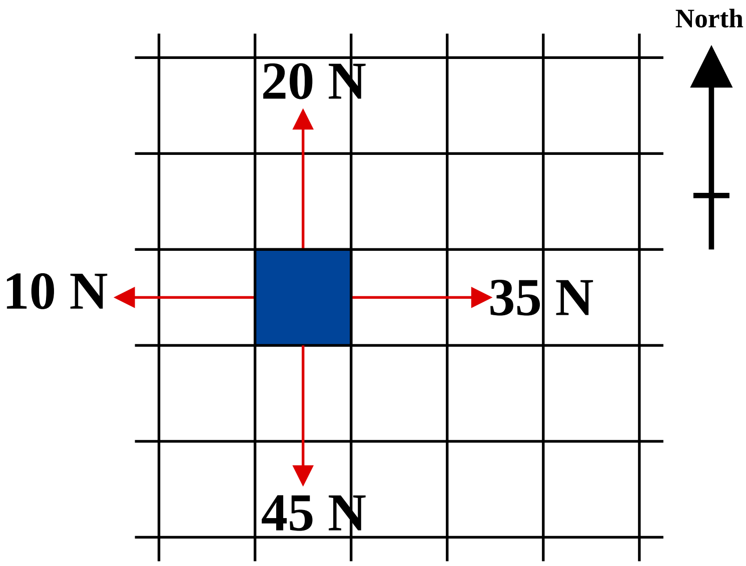
(v) At what angle to the positive x-axis direction does the equilibrant act ?

Question 2

A wooden box rests on a sheet of smooth ice.



Seen from above, the four horizontal forces acting on the wooden box are,

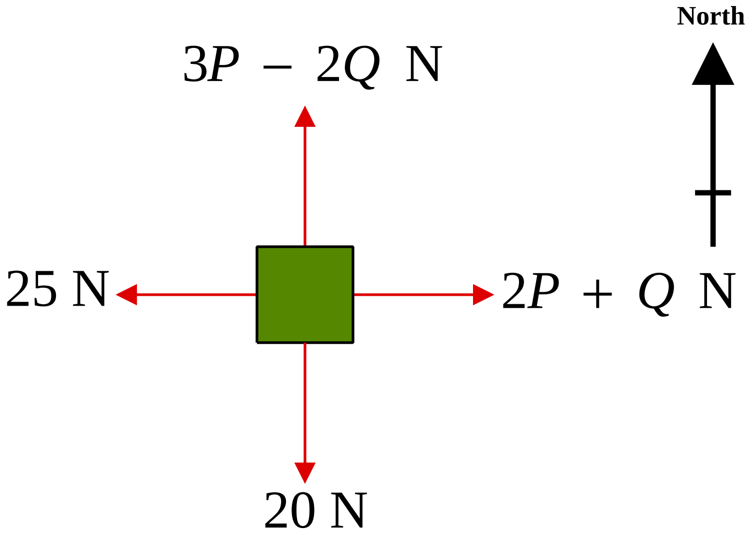


(i) Determine the magnitude of the resultant force.

(ii) In what direction does the resultant force act ?
Give your answer as a compass direction.

Question 3

Another wooden box on another sheet of smooth ice is moving with constant velocity north-east with four forces acting upon it, as shown below,



Find the value of P and the value of Q

Question 4

Three forces, $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are in equilibrium.

Given that, in Newtons, the forces are,

$$\boldsymbol{u} = \begin{pmatrix} a \\ 2b \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} -2a \\ -b \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$$

find the value of the constant a and the value of the constant b

Question 5

Consider the following three vectors which represent forces measured in Newtons;

$$\boldsymbol{u} = \begin{pmatrix} a + 1 \\ a \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} 1 \\ 1 - a \end{pmatrix} \qquad \boldsymbol{w} = \begin{pmatrix} a - 1 \\ 1 + a \end{pmatrix}$$

where a is a constant.

By summing the vectors, or otherwise, prove that if these three forces were to act of a particle then that particle could ***not*** be in equilibrium.

Question 6

Philomena Cunk, in an interview with scientist Brian Cox, argues that there is no need for angels to be pushing the Moon along by beating their wings because the Sun, Moon and the Earth are in fact stationary relative to each other in the sky. She claims that it is the rotation of the Earth in its stationary position that gives the illusion of the Moon moving across the Earth's sky.

Give a brief but convincing argument backed up by observational evidence to explain why Philomena must be wrong.

Question 7

In this question, **i** represents the unit vector due east, and **j** represents the unit vector due north.

A particle is acted upon by three forces, ***u***, ***v*** and ***w***, which, measured in Newtons, are,

$$\boldsymbol{u} = -2\mathbf{i} + \mathbf{j} \qquad \boldsymbol{v} = 5\mathbf{i} + 2\mathbf{j} \qquad \boldsymbol{w} = -\mathbf{i} - 4\mathbf{j}$$

Work out,

(i) the resultant vector

(ii) the magnitude of the resultant vector

(iii) the bearing of the resultant vector

(iv) the equilibrant

Question 8

A particle is acted upon by two forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ which, measured in Newtons, are,

$$\boldsymbol{F_1} = 3\mathbf{i} - 2\mathbf{j} \qquad\qquad \boldsymbol{F_2} = a\mathbf{i} + 3a\mathbf{j}$$

where a is a positive constant and where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors in the x-axis and y-axis directions respectively.

(i) Find the angle between $\boldsymbol{F_2}$ and $\mathbf{i}$

[2 marks]

The resultant of $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ is $\boldsymbol{R}$

(ii) Given that $\boldsymbol{R}$ is parallel to $18\mathbf{i} + 10\mathbf{j}$ find the value of a

[4 marks]

Question 9

Three forces $\boldsymbol{F_1}$, $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ act on a particle P.

Expressed as vectors, in newtons, the forces are,

$$\boldsymbol{F_1} = \begin{pmatrix} -7 \\ -4 \end{pmatrix} \qquad \boldsymbol{F_2} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \qquad \text{and} \qquad \boldsymbol{F_3} = \begin{pmatrix} a \\ b \end{pmatrix}$$

where a and b are constants.

(i) Given that P is in equilibrium, find the value of a and the value of b

[3 marks]

The force $\boldsymbol{F_1}$ is now removed.

The resultant of $\boldsymbol{F_2}$ and $\boldsymbol{F_3}$ is $\boldsymbol{R}$

(ii) Determine the magnitude of $\boldsymbol{R}$

[2 marks]

(iii) Find the angle, to the nearest degree, that the direction of $\boldsymbol{R}$ makes with the horizontal.

[3 marks]