Lesson 4

A-Level Applied Mathematics

Mechanics : Dynamics I : Year 1

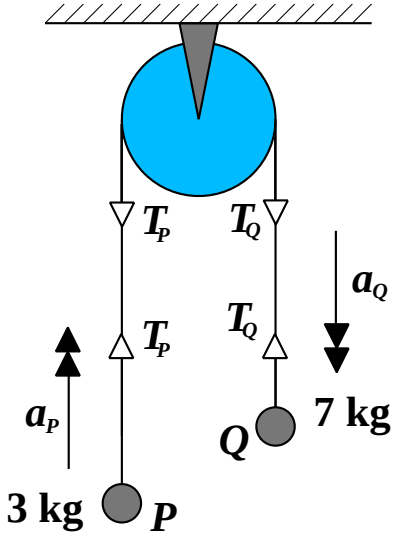
4.1 Pulleys

If a heavy item has to be lifted a pulley may be employed to make the task easier. Even with just one pulley, a person can lift far more than would otherwise be the case. This is because a part of their own weight is assisting in the lift.

The classic use of a pulley is to get a piano into an upstairs room in a house by hoisting it up the outside of the house (it's too big to go up an internal staircase). This real life application is often used for comic effect in cartoons and movies.

4.2 Pulley Example

Particles P and Q are of mass 3 kg and 7 kg respectively. They are attached to the ends of a light inextensible string which passes over a smooth fixed pulley. The system is released from rest.



(i) Find the constant acceleration of the system and the tension in the string.

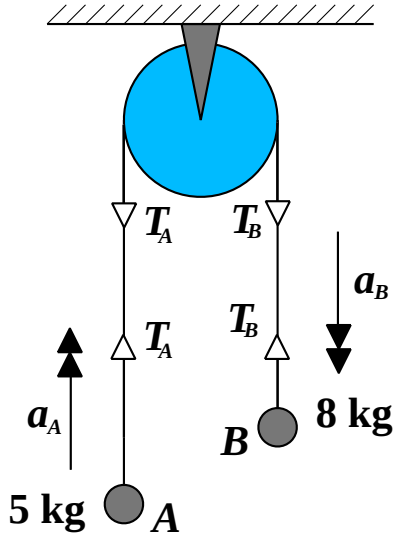
(ii) Assuming that P does not reach the pulley, and that Q does not reach the ground, what distance will Q travel in the first 3 seconds of motion ?

(iii) If the string was not 'light', would the acceleration still be constant ? Explain your answer.

4.3 Exercise

Question 1

Two particles A and B of masses 5 kg and 8 kg respectively are connected by a light inextensible string which passes over a small smooth fixed pulley. The particles are released from rest with the string taut.



(i) Find the tension in the string

When B has travelled 2 metres it strikes the ground and immediately comes to rest.

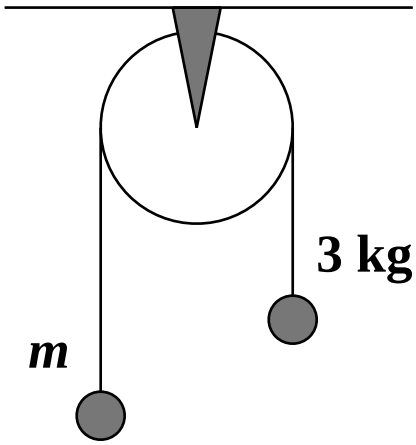
(ii) Find the speed of B when it hits the ground.

(iii) Assuming that A does not hit the pulley, find the greatest height that A reaches above its initial position.

Question 2

Two particles are connected by a light inextensible string which passes over a smooth fixed pulley. One mass is 3 kg, the other is unknown, m kg where $m < 3$. The system is released from rest with the string taut.

The acceleration of each particle is 2.2 m s^{-2}



(a) Determine the tension in the string

(b) Find the value of m

(c) What is the force exerted by the system on the ceiling when,

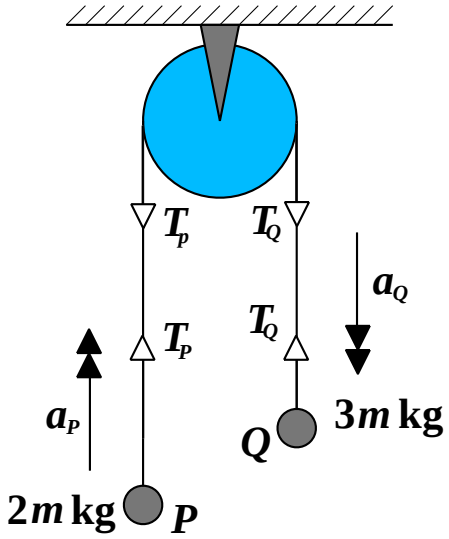
(i) the system was at rest, initially (when the pulley was 'locked' and so unable to turn) ?

(ii) the system was in motion ?

(d) State how you have used the modelling assumption that the pulley is smooth

Question 3

Particles P and Q of masses $2m$ and $3m$ are attached to the ends of a light inextensible string. The string passes over a small smooth fixed pulley and the masses hang with the string taut. The system is released from rest.



(a) **(i)** Write down an equation of motion for P

(ii) Write down an equation of motion for Q

(b) Find the acceleration of each mass

Given that the force exerted on the ceiling by the system when in motion is 117.6 N,

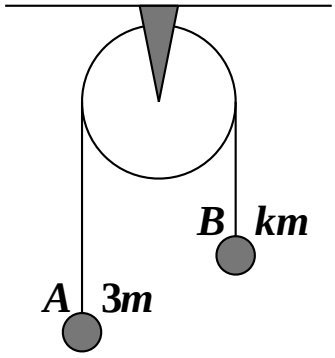
(c) Find the tension in the string

(d) Determine the value of m

(e) Find the distance moved by Q in the first 4 seconds, assuming that P does not reach the pulley.

(f) State how you have used the fact that the string is inextensible in your calculations

Question 4



Two particles A and B have masses of $3m$ and km respectively, where $k > 3$. They are connected by a light inextensible string which passes over a smooth fixed pulley. The system is released from rest with the string taut and the hanging parts of the string vertical.

While the particles are moving freely, A as an acceleration of magnitude $\frac{2}{5}g$

(a) Find, in terms of m and g the tension in the string.

[3 marks]

(b) State why B also has an acceleration of magnitude $\frac{2}{5}g$

[1 mark]

(c) Find the value of k

[4 marks]

(d) State how you have used the fact that the string is light

[1 mark]