Applied A-Level Mathematics
Mechanics
Year 2

MOMENTS
II

Ten nails balanced on the head of a single nails.
The most nails balanced on the head of a single nail is 349 and was achieved by Alexander Bendikov in Sevastopol, Russia, on 26th May 2018
(Guinness World Records, April 2020)

M O M E N T S I I

Lesson 1

A-Level Applied Mathematics
Mechanics : Moments II : Year 2

1.1 Introduction

In this second look at the topic of Moments, ideas previously explored are developed further using the key ideas of resolving forces and friction. The mechanical systems considered are still simple but, even so, are often encountered in everyday life. These include such items as the metalwork supporting a pub's hanging sign, a ladder against a wall, and a plank of wood resting against an oil barrel that's on its side. In such situations, unlike those considered previously, the main rod may not be horizontal or the forces acting on the rod may not be at right angles to the rod.

1.2 A Hanging Basket

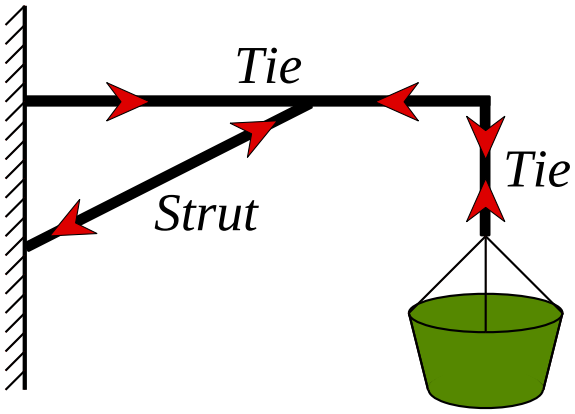
Most people, when looking at a hanging basket, focus on the plant in the basket. To a mechanical engineer, however, the supporting structure is of more interest. Notice that the main element in the metalwork is a horizontal rod to which the basket is attached at the end furthest from the wall. Expecting this rod to simply stick out of the vertical wall and, without bending, hold up a heavy basket would require a heavier and thicker rod than is being employed here. There is a second rod, called a strut, that is supporting the main rod, from below. It's because this second rod has internal forces that are thrusting outward at each end that it's called a strut.
Previous work involving, for example, a tow bar between a car and a caravan, dealt with a tie, the tow bar; a rod with an internal inward force at either end; a tension.

1.3 Struts and Ties

This is a strut.
Its internal forces are pushing outward at either end.
The rod is resisting being compressed, that is, it's resisting being made shorter.
The forces within the rod are referred to as thrusts.

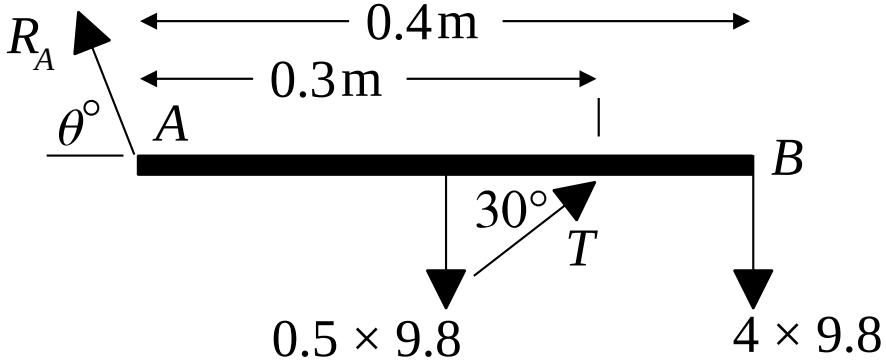
This is a tie.
Its internal forces are pulling inward at either end.
The rod is resisting being stretched, that is, it's resisting being made longer.
The forces within the rod are referred to as tensions.

Here is a diagram of the hanging basket showing some of the struts and ties.



1.4 Force at an Angle

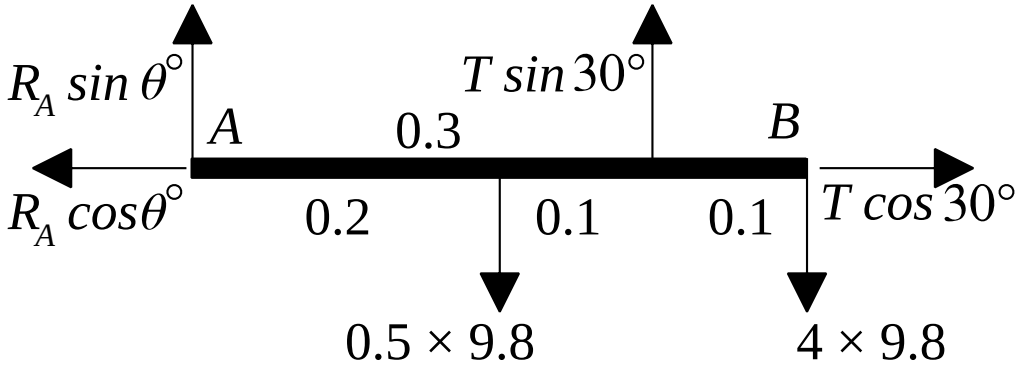
With the difference between a strut and a tie understood a diagram showing the forces acting on the horizontal rod holding up the hanging basket can now be drawn.



Notice the force from the strut is acting at an angle, as is the force exerted on the rod by the wall, R_A. The angle between strut and rod is easy to physically measure, it's 30°, and the length to where the strut meets the rod is also easily measured, it's 0.3 metres along the 0.4 metre rod from *A*.

For a mechanical engineer, the problem now is to determine the thrust in the strut and the force of the wall on the rod. It is good practice for the joint at the wall not to be under unnecessarily large forces and so it is assumed that the joint there can be modelled as a smooth hinge. The rod is assumed to be uniform.

The solution begins having already resolved the thrust, T, and the reaction R_A into component parts perpendicular and parallel to the rod, thus obtaining the following working diagram,



Determine **(i)** The thrust, T, in the strut.

(ii) The magnitude of the reaction, R_A, of the wall on the rod.

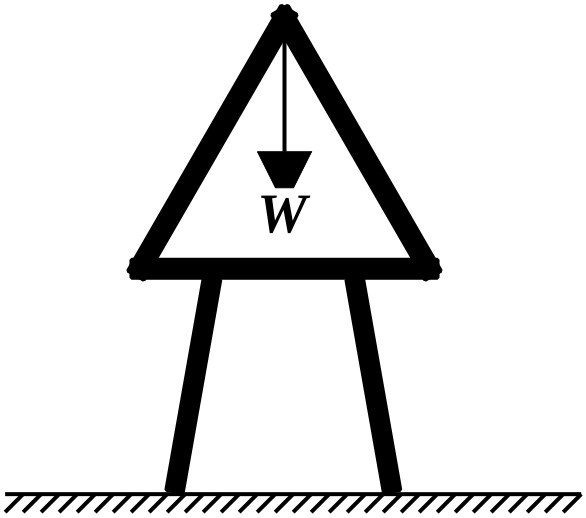
(iii) The angle, θ, the reaction R_A makes to the horizontal.

Teaching Video : http://www.NumberWonder.co.uk/Video/v9084(1).mp4

1.5 Exercise

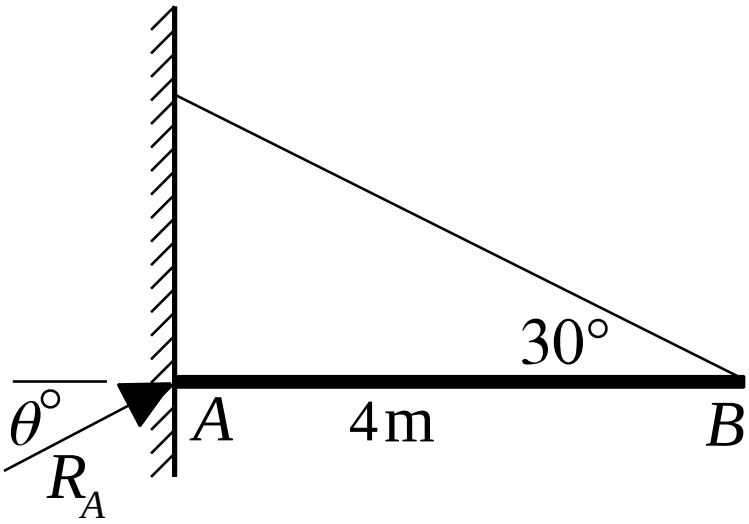
Question 1

Five wooden beams are joined together to form a structure from which a weight, *W*, can be raised and lowered. Where each beam is connected to another a single bolt is used, effectively forming a smooth hinge. The structure is concreted into the ground. On the diagram next to each of the five beams label if it is acting as a strut or a tie.



Question 2

A nonuniform rod *AB* has a weight of 100 N and length 4 metres.
It is freely hinged to a vertical wall at *A*, and held in horizontal equilibrium by a cable attached at an angle of 30° to end *B*.
The tension in the cable is 80 N.



From an inspection of the hinge at end *A*, it is clear that the wall is exerting a force on the rod in the approximate direction indicated on the diagram by R_A.

(i) When the 80 N force in the cable is described as a “tension” (rather than a “thrust”) does this tell you that the supporting cable is a strut or a tie ?

(ii) Draw a “working diagram” showing all significant forces that are acting on the rod. Any forces that are not parallel or perpendicular to the rod should be resolved into component parts which are.

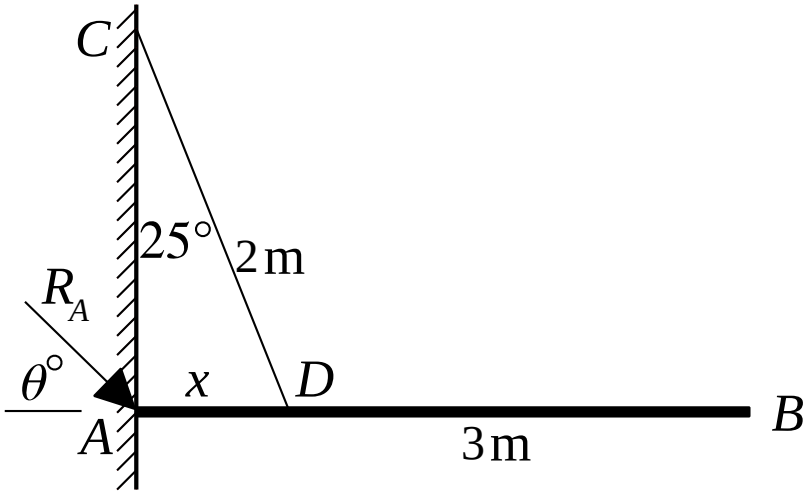
(iii) By taking moments about *A*, find the distance of the centre of mass of the nonuniform rod from the wall.

(iv) By balancing forces vertically obtain one equation in terms of R_A and θ.

(v) By balancing forces horizontally obtain another equation in terms of R_A and θ.

(vi) Solve your part (iv) and (v) equations simultaneously to find R_A and θ.

Question 3



A beam *AB* of mass 18 kg and length 3 m is smoothly hinged to a vertical wall at *A*.
The beam is held in equilibrium in a horizontal position by a wire of length 2 m.
One end of the wire is fixed to a point *C* on the wall vertically above *A*.
The other end of the wire is fixed to a point *D* on the beam such that angle *ACD* is 25°.
The beam is modelled as a uniform rod and the wire as a light inextensible string.
From looking at the hinge at end *A*, it is clear that the wall is exerting a force on the rod in the approximate direction indicated on the diagram by R_A.

(i) Draw a "working diagram" showing all significant forces that are acting on the rod. Any forces that are not parallel or perpendicular to the rod should be resolved into component parts which are.

(ii) Use simple trigonometry to calculate the length *AD*, marked *x* in the diagram.

(iii) By taking moments about *A*, find the tension in the tie *DC*.

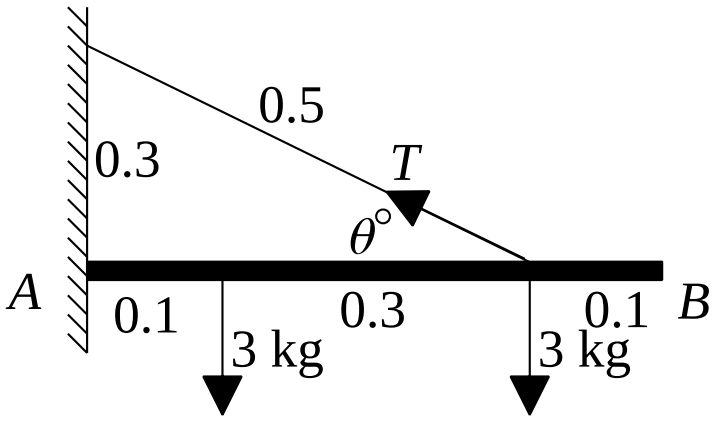
(iv) By balancing forces vertically obtain an equation in terms of R_A and θ.

(v) By balancing forces horizontally obtain another equation in terms of R_A and θ.

(vi) Solve your part (iv) and (v) equations simultaneously to find R_A and θ.

Question 4

The Bath Sweet Shop has a 6 kg sign suspended centrally from two points 0.3 metres apart from an 0.5 metre metal rod. This horizontal uniform rod has a mass of 2 kg and is supported by a tie which is also of length 0.5 metres. The rod is attached to the wall by a smooth hinge which is 0.3 metres vertically below where the tie is attached to the wall. The tie is attached to the rod 0.4 metres along the rod where it makes an angle of θ to the rod. Some, but not all, of this information is shown on the diagram below,



Determine
(i) The angle marked θ.
(ii) The tension T in the tie supporting the rod.
(iii) The magnitude and the direction of the force applied by the wall on the rod at end A.