Lesson 10

Further A-Level Pure Mathematics : Core 1

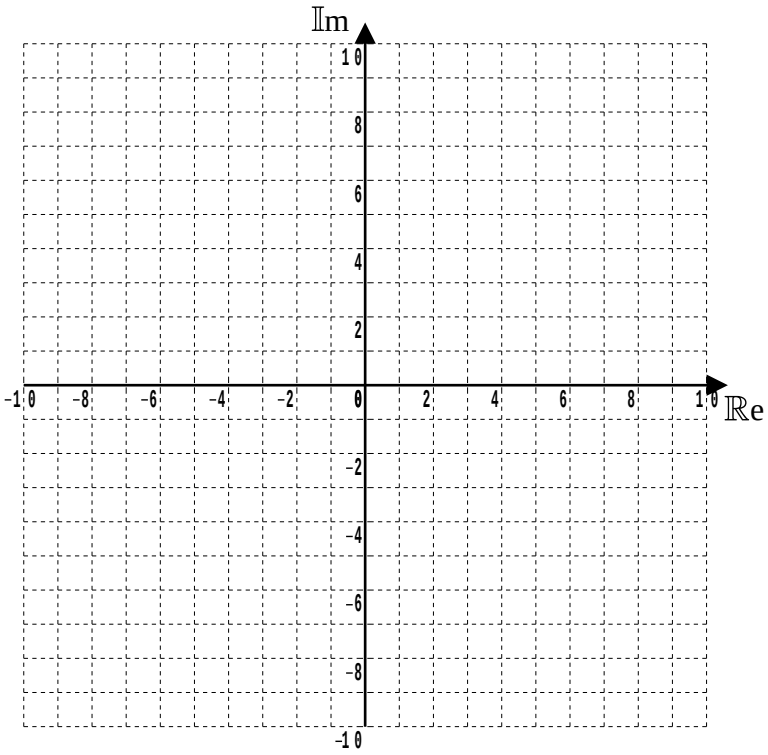
Complex Numbers I

10.1 Regions and Argand Diagrams

On the Argand diagram, shade in the region specified by,

$$\{\, 3 < |z - 3 + 5\mathrm{i}| \leqslant 5 \,\} \cap \left\{ -\frac{\pi}{2} \leqslant arg\,(z - 3 + 5\mathrm{i}) < \frac{3\pi}{4} \right\}$$

Teaching Video : http://www.NumberWonder.co.uk/Video/v9085(10).mp4



[4 marks]

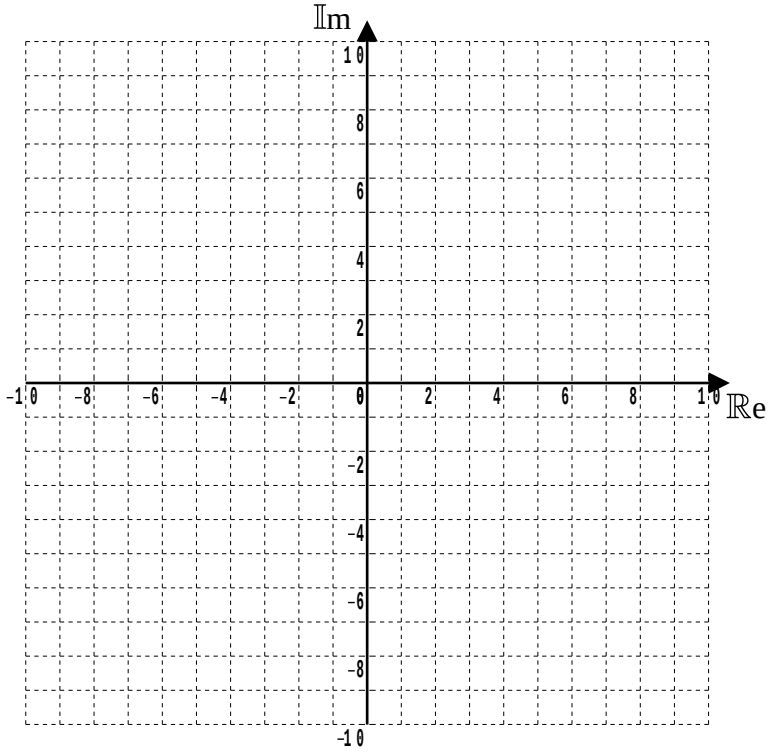
10.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 32

Question 1

(i) On the Argand diagram, shade in the region, *R*, specified by,

$$\{ 3 \leqslant |z + 3 - 4i| < 5 \} \cap \left\{ -\frac{\pi}{3} < arg\,(z + 3 - 4i) \leqslant \frac{\pi}{3} \right\}$$



[4 marks]

(ii) Show how to use the formula $Arc\ length = r\theta^c$ to determine the ***exact*** perimeter of the region *R*

[2 marks]

(iii) Show how to use the formula $Sector\ area = \frac{1}{2} r^2 \theta^c$ to determine the ***exact*** area of the region *R*

[2 marks]

Question 2

Further A-Level SAM Question for Core Pure Mathematics 2, June 2020, Q6 (b) (i)

The set of points A is defined by

$$A = \left\{ z \ : \ 0 \leqslant arg\, z \leqslant \frac{\pi}{3} \right\} \cap \left\{ z \ : \ |z - 4 - 3\mathrm{i}| \leqslant 5 \right\}$$

Show, by shading on an Argand diagram, the set of points A

[2 marks]

Question 3

The complex number z is represented by a point P on an Argand diagram.

Given that $|z + 4 - 4\mathrm{i}| \leqslant 4$ and $\frac{\pi}{2} \leqslant arg(z) \leqslant \frac{3\pi}{4}$ shade the locus of P

[2 marks]

Question 4

Further A-Level Examination Question from June 2003, FP2, Q1 (edited)

(a) On the same Argand diagram sketch the loci given by the following equations,

$$|z - 1| = 1 \qquad arg(z + 1) = \frac{\pi}{12} \qquad arg(z + 1) = \frac{\pi}{2}$$

[4 marks]

(b) Shade on your diagram the region for which

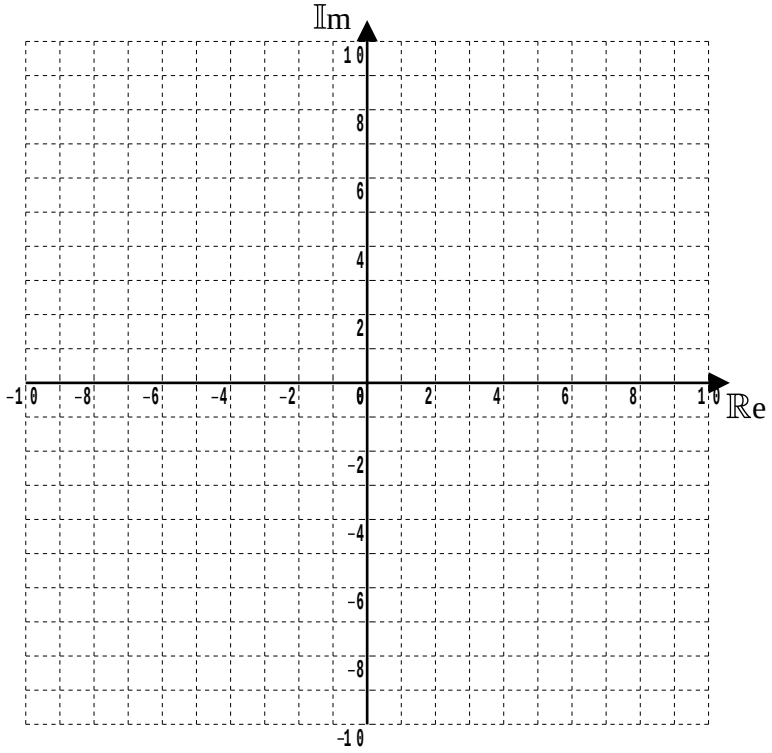
$$|z - 1| \leqslant 1 \qquad \text{and} \qquad \frac{\pi}{12} \leqslant arg(z + 1) \leqslant \frac{\pi}{2}$$

[1 mark]

Question 5

(i) On the Argand diagram, shade in the region, *R*, specified by,

$$\{ z \in \mathbb{C} : 3 < |z| \leqslant 5 \} \cap \{ z \in \mathbb{C} : |z| \leqslant |z - 6i| \}$$



[4 marks]

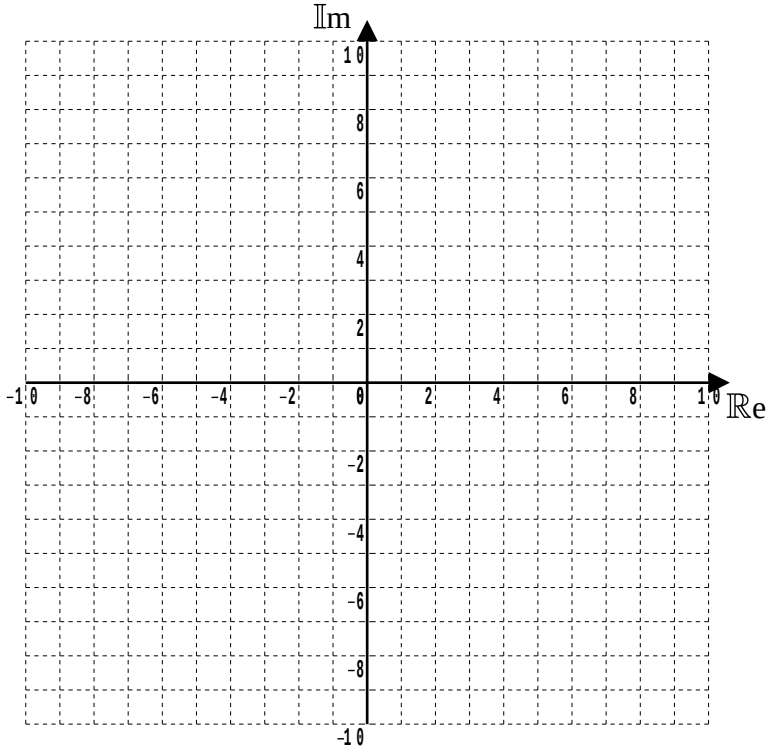
(ii) Determine the area of the region *R*, correct to three significant figures.

[3 marks]

Question 6

(i) On the Argand diagram, shade in the region, R, specified by,

$$\{ z \in \mathbb{C} : |z| \leqslant 6 \} \cap \left\{ z \in \mathbb{C} : 0 \leqslant arg(z + 6) \leqslant \frac{\pi}{4} \right\}$$



[4 marks]

(ii) Determine the ***exact*** perimeter of the region R

[2 marks]

(iii) Determine the ***exact*** area of the region R

[2 marks]