9.1 A Catalogue of Loci

Over the last few lessons, a first steps towards developing a catalogue of loci on Argand diagrams have been taken. It has thus far only involved two shapes, two objects familiar from previous study.

Generalising previous specific examples gives a catalogue with two entries;

- $|z - (a + b\text{i})| = r$ where $a, b, r \in \mathbb{R}$, and $r > 0$

  A circle with centre (a, b) and radius r

- $|z - (a + b\text{i})| = |z - (c + d\text{i})|$ where $a, b, c, d \in \mathbb{R}$

  A line that is the perpendicular bisector of the line segment between the points (a, b) and (c, d)

9.2 The Argument as a Locus

This lesson a third entry to the catalogue will be developed.

Given that $arg(z - 3 + 2\text{i}) = \dfrac{2\pi}{3}$ give the Cartesian equation of the locus and sketch the locus of z on an Argand diagram.

Teaching Video : http://www.NumberWonder.co.uk/Video/v9085(9).mp4

[4 marks]

9.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 33

Question 1

Given that $arg(z - 2 + 3\text{i}) = \frac{3\pi}{4}$ give the Cartesian equation of the locus and sketch the locus of z on an Argand diagram.

[4 marks]

Question 2

Given that $arg(z - 3 - 4\text{i}) = -\frac{2\pi}{3}$ give the Cartesian equation of the locus and sketch the locus of z on an Argand diagram.

[4 marks]

Question 3

Further A-Level Examination Question from June 2005, FP2, Q9

A complex number z is represented by the point P in the Argand diagram.

Given that,

$$|z - 3i| = 3$$

(a) sketch the locus of P

[2 marks]

(b) Find the complex number z which satisfies both

$$|z - 3i| = 3 \quad \text{and} \quad arg(z - 3i) = \frac{3\pi}{4}$$

[4 marks]

Question 4

Given that z satisfies,

$$\left| z + \sqrt{3}\,\mathrm{i} \right| = 3$$

(a) sketch the locus of z on an Argand diagram

[2 marks]

(b) find z that satisfies both $\left| z + \sqrt{3}\,\mathrm{i} \right| = 3$ and $arg(z) = \dfrac{\pi}{6}$

[4 marks]

Question 5

Given that

$$arg(z + 4) = \frac{\pi}{3}$$

(a) sketch the locus of z on an Argand diagram

[2 marks]

(b) find the minimum value of $|z|$ for points on this locus

[4 marks]

Question 6

Sketch on the same Argand diagram the locus of points satisfying,

(a) $|z - 2\mathrm{i}| = |z - 8\mathrm{i}|$

[2 marks]

(b) $arg(z - 2 - \mathrm{i}) = \dfrac{\pi}{4}$

[3 marks]

The complex number z satisfies both

$$|z - 2\mathrm{i}| = |z - 8\mathrm{i}| \quad \text{and} \quad arg(z - 2 - \mathrm{i}) = \frac{\pi}{4}$$

(c) Use your answers to parts (a) and (b) to find the value of z

[2 marks]