

Lesson 9

Further A-Level Pure Mathematics : Core 1 Complex Numbers I

9.1 A Catalogue of Loci

Over the last few lessons, a first steps towards developing a catalogue of loci on Argand diagrams have been taken. It has thus far only involved two shapes, two objects familiar from previous study.

Generalising previous specific examples gives a catalogue with two entries;

- $|z - (a + bi)| = r$ where $a, b, r \in \mathbb{R}$, and $r > 0$

A circle with centre (a, b) and radius r

- $|z - (a + bi)| = |z - (c + di)|$ where $a, b, c, d \in \mathbb{R}$

A line that is the perpendicular bisector of the line segment between the points (a, b) and (c, d)

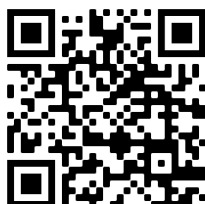
9.2 The Argument as a Locus

This lesson a third entry to the catalogue will be developed.

Given that $\arg(z - 3 + 2i) = \frac{2\pi}{3}$ give the Cartesian equation of the locus

and sketch the locus of z on an Argand diagram.

Teaching Video : [http://www.NumberWonder.co.uk/Video/v9085\(9\).mp4](http://www.NumberWonder.co.uk/Video/v9085(9).mp4)



9.3 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 33

Question 1

Given that $\arg(z - 2 + 3i) = \frac{3\pi}{4}$ give the Cartesian equation of the locus

and sketch the locus of z on an Argand diagram.

[4 marks]

Question 2

Given that $\arg(z - 3 - 4i) = -\frac{2\pi}{3}$ give the Cartesian equation of the locus

and sketch the locus of z on an Argand diagram.

[4 marks]

Question 3

Further A-Level Examination Question from June 2005, FP2, Q9

A complex number z is represented by the point P in the Argand diagram.

Given that,

$$|z - 3i| = 3$$

- (a) sketch the locus of P

[2 marks]

- (b) Find the complex number z which satisfies both

$$|z - 3i| = 3 \quad \text{and} \quad \arg(z - 3i) = \frac{3\pi}{4}$$

[4 marks]

Question 4

Given that z satisfies,

$$|z + \sqrt{3}i| = 3$$

- (a) sketch the locus of z on an Argand diagram

[2 marks]

- (b) find z that satisfies both $|z + \sqrt{3}i| = 3$ and $\arg(z) = \frac{\pi}{6}$

[4 marks]

Question 5

Given that

$$\arg(z + 4) = \frac{\pi}{3}$$

- (a) sketch the locus of z on an Argand diagram

[2 marks]

- (b) find the minimum value of $|z|$ for points on this locus

[4 marks]

Question 6

Sketch on the same Argand diagram the locus of points satisfying,

(a) $|z - 2i| = |z - 8i|$

[2 marks]

(b) $\arg(z - 2 - i) = \frac{\pi}{4}$

[3 marks]

The complex number z satisfies both

$$|z - 2i| = |z - 8i| \quad \text{and} \quad \arg(z - 2 - i) = \frac{\pi}{4}$$

(c) Use your answers to parts (a) and (b) to find the value of z

[2 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk