**Year 1 Pure Mathematics Examination Revision**
**Health Check N° 2**

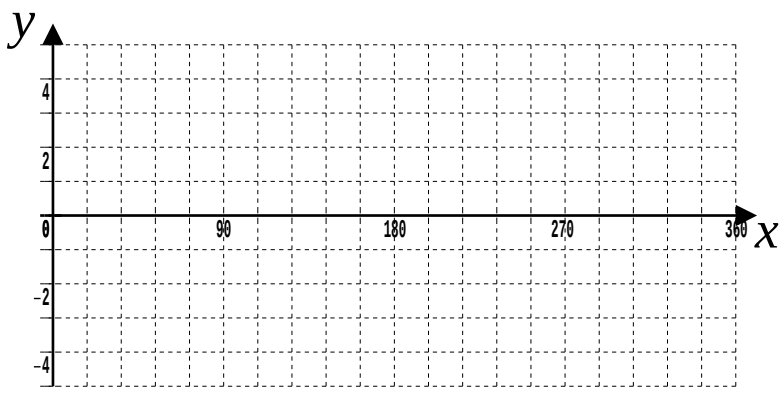

I went to the Library to get a medical book on abdominal pain. Somebody had ripped the appendix out...

*Any solution based entirely on graphical or numerical methods is not acceptable*
Marks Available : 50

**Question 1**

Given a curve, $y = f(x)$, if $y$ is replaced with $\frac{y}{3}$ all distances from the $x$-axis are tripled and if $x$ is replaced with $2x$ all distances from the $y$-axis halve.

Use these facts to sketch the graph of $\frac{y}{3} = sin(2x)$ on the grid below.




**[ 3 marks ]**

**Question 2**

**( i )** Show that $\frac{1 - tan^2 x}{1 + tan^2 x} = 2 cos^2 x - 1$

**[ 6 marks ]**

**( ii )** Hence solve $\frac{1 - tan^2 x}{1 + tan^2 x} = \frac{1}{8}$ over the interval $0° \leqslant x \leqslant 360°$

**[ 5 marks ]**

**Question 3**

**( i )** On the same graph sketch the curve $y = (x - 1)^2 (x + 1)^2$ and the straight line $y = 1$, paying particular attention to any points where an axis is touched or crossed.

**[ 5 marks ]**

**( ii )** Find all solutions to the equation, $(x - 1)^2 (x + 1)^2 = 1$

**[ 4 marks ]**

**( iii )** Solve $\left( \frac{1}{\cos x} - 1 \right)^2 \left( \frac{1}{\cos x} + 1 \right)^2 = 1, \quad 0 \leqslant x \leqslant 360, \; x \neq 90, \; 270$

**[ 4 marks ]**

**Question 4**

To translate any curve by the vector $\begin{pmatrix} a \\ b \end{pmatrix}$

- replace $x$ with $x - a$
- replace $y$ with $y - b$

**( i )** Given that the equation of a circle centre ( 0, 0 ), radius $r$ is $x^2 + y^2 = r^2$

deduce the equation of a circle of radius 13 that has been translated $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$

**[ 3 marks ]**

**( ii )** The parabola $y = x^2$ is to be translated $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

What is the equation of the translated parabola ?

Give your answer in the form $y = ax^2 + bx + c$ where $a$, $b$ and $c$ are integers the values of which you have determined.

**[ 4 marks ]**

**( iii )** The inverse proportion graph, $y = \frac{1}{x}$ is translated so that the asymptotes are at $x = 5$ and $y = 1$.

Find the equation of the transformed graph in the form $y = \frac{ax + b}{cx + d}$ where $a$, $b$, $c$ and $d$ are integers, the values of which you have determined.

**[ 5 marks ]**

**Question 5**

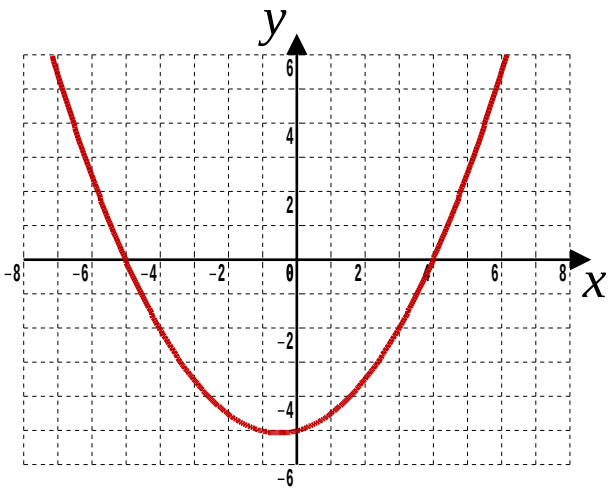
In $\triangle ABC$, $AB = 16$ cm, $AC = 13$ cm, $\angle ABC = 50°$ and $\angle BCA = x°$

Find the two possible values for $x$, giving your answers to one decimal place.

**[ 4 marks ]**

**Question 6**

The parabola shown below crosses the $x$-axis at $(-5,\ 0)$ and $(4,\ 0)$ and it crosses the $y$-axis at $(0, -5)$




**( i )** Determine the equation of the parabola in the form $y = f(x)$

**[ 4 marks ]**

**( ii )** Solve the related equation $f(x - 3) = 0$

**[ 3 marks ]**