

**Year 1 Pure Mathematics Examination Revision**  
**Health Check N° 3**



When you get a bladder infection, urine trouble...

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 50

**Question 1**

Show that,  $\cos^4 x - \sin^4 x = 1 - 2 \sin^2 x$

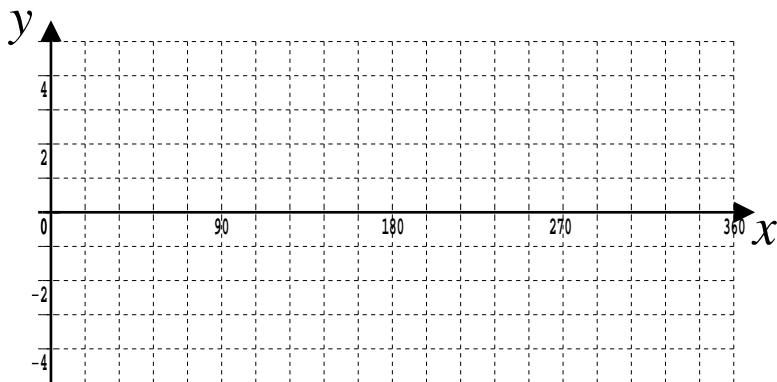
[ 4 marks ]

**Question 2**

Given a curve,  $y = f(x)$ , if  $y$  is replaced with  $\frac{y}{4}$  all distances from the  $x$ -axis

are quadrupled and if  $x$  is replaced with  $(x - 36^\circ)$  the graph translates  $\begin{pmatrix} 36 \\ 0 \end{pmatrix}$

Use these facts to sketch the graph of  $y = 4 \sin(x - 36^\circ)$  on the grid below.



[ 3 marks ]

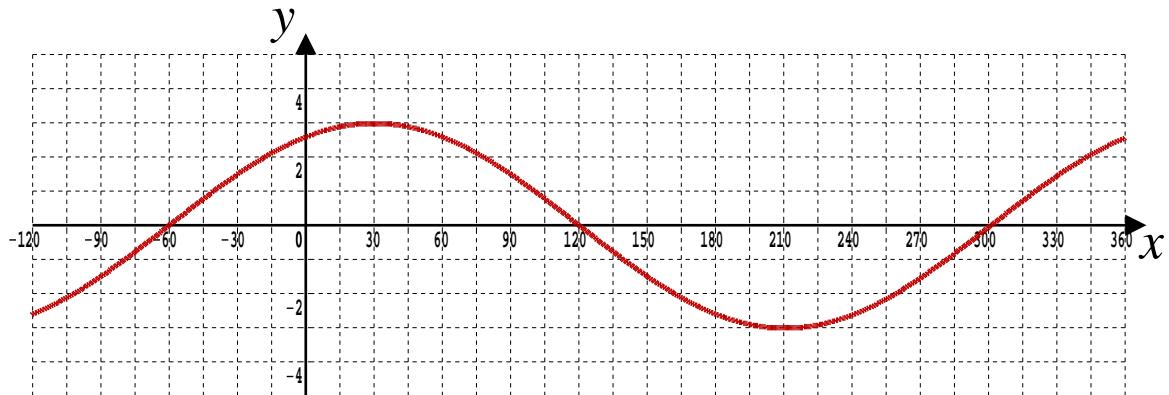
**Question 3**

Expand the brackets and simplify,  $\frac{(1 + x)^8 - (1 - x)^8}{16}$

[ 5 marks ]

**Question 4**

The curve shown has an equation of the form  $y = a \sin(x + b)$  for some constants  $a$  and  $b$  and where  $x$  is measured in degrees.



(i) Find the value of  $a$  giving a clear reason for your answer.

[ 2 marks ]

(ii) Find the value of  $b$  giving a clear reason for your answer.

[ 2 marks ]

(iii) Consider solving an equation of the form,

$$a \sin(x + b) = k,$$

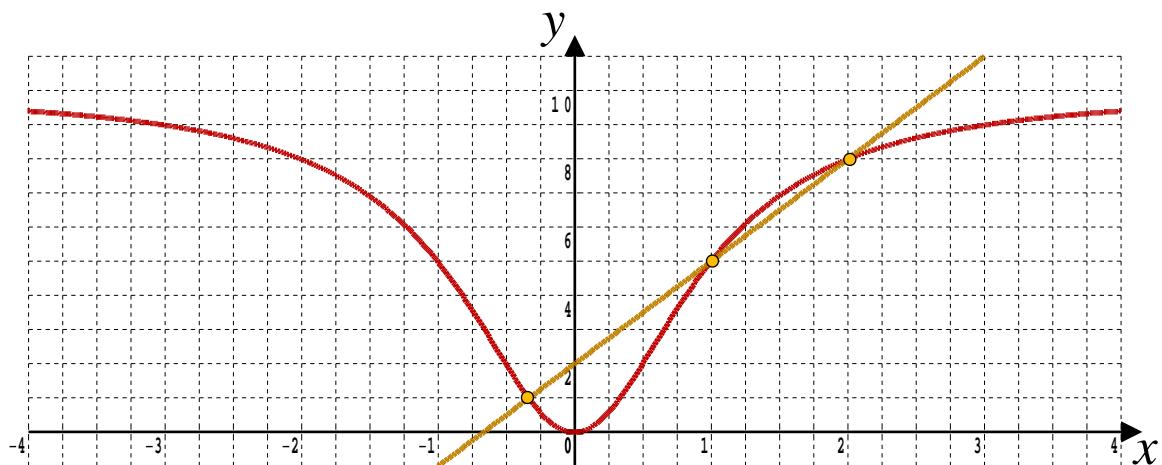
where  $k$  is a positive constant.

State the range of possible values for  $k$  for which the equation has solutions.  
Again, give a reason for your answer.

[ 2 marks ]

**Question 5**

Peyton has graphed the curve  $y = \frac{10x^2}{x^2 + 1}$  and the straight line  $y = 3x + 2$



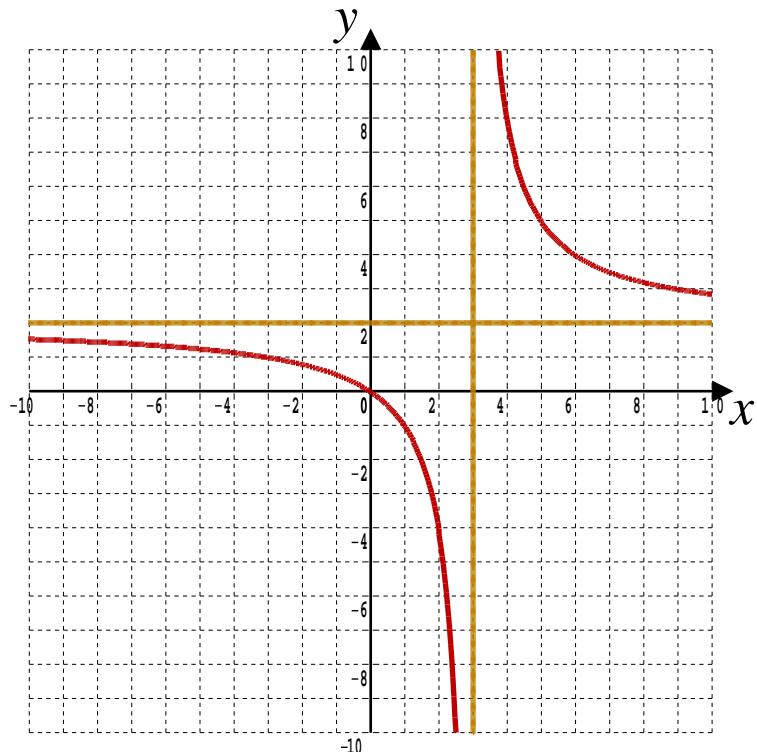
Use algebra to determine the exact coordinates of all three points of intersection.

[ 6 marks ]

### Question 6

The graph of inverse proportionality has been translated such that its asymptotes are  $x = 3$  and  $y = 2$ .

Furthermore the translated graph passes through the origin.



Determine the equation of the translated curve in the form  $y = \frac{ax + b}{cx + d}$   
where  $a, b, c$  and  $d$  are integers, the values of which you have determined.

[ 6 marks ]

**Question 7***Oxford University Mathematics Admission Test, 2017, Q(B)*

(i) By completing the square, or otherwise, find the minimum value of,

$$f(x) = 9 \cos^4 x - 12 \cos^2 x + 7$$

**[ 6 marks ]**

(ii) Find the four values of  $x$  between  $0^\circ$  and  $360^\circ$  that give rise to the minimum value of,

$$f(x) = 9 \cos^4 x - 12 \cos^2 x + 7$$

Give your answers to one decimal place.

**[ 4 marks ]**

### Question 8

From applying the binomial theorem it is known that the first three terms of,

$$\left(1 + \frac{x}{3}\right)^{18} = 1 + 6x + 17x^2 + \dots$$

Use this expansion to find an estimate of the value of  $1.01^{18}$ , giving your answer to 4 decimal places.

[ 4 marks ]

### Question 9

*Oxford University Mathematics Admission Test, 2010, Q(A)*

Find the values of  $k$  for which the straingh line with equation  $y = kx$  intersects the parabola with equation  $y = (x - 1)^2$

[ 6 marks ]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@gmail.com