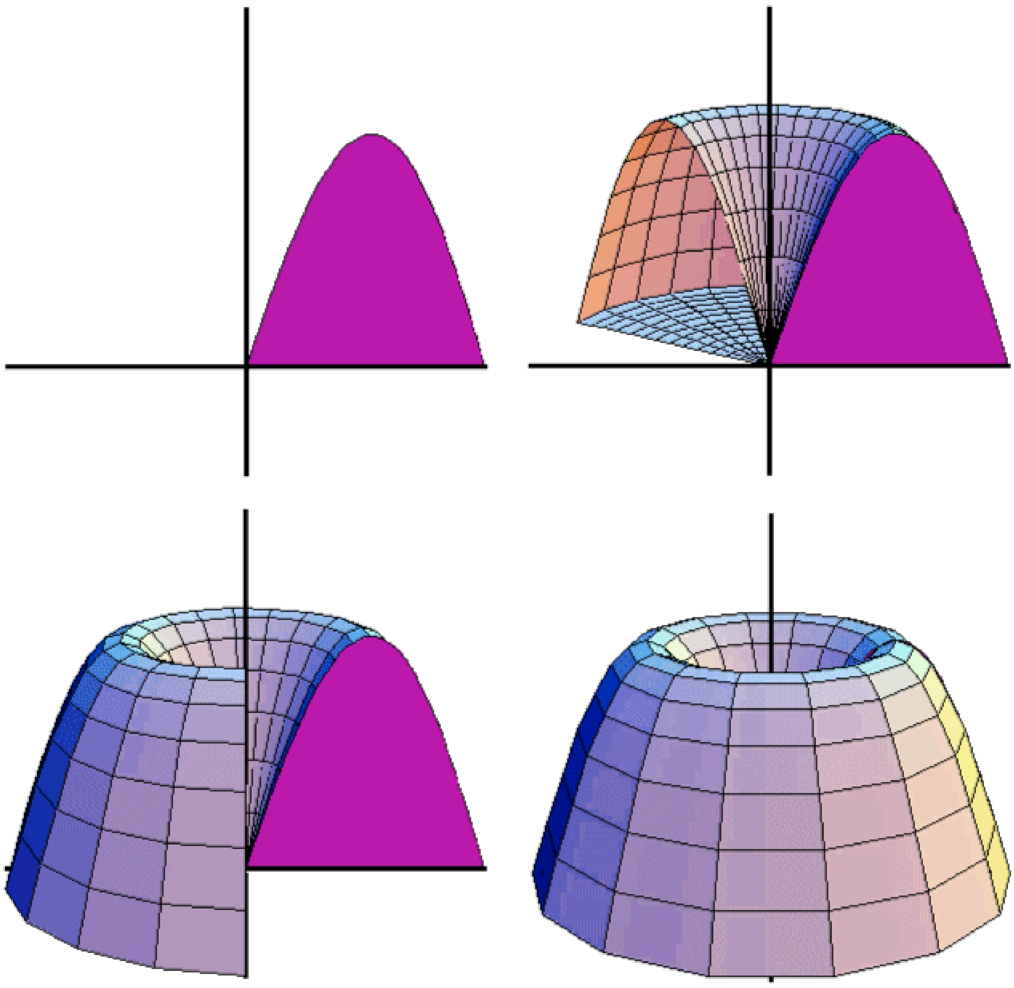
Further Pure A-Level Mathematics
Compulsory Course Component
Core 1

VOLUMES of REVOLUTION

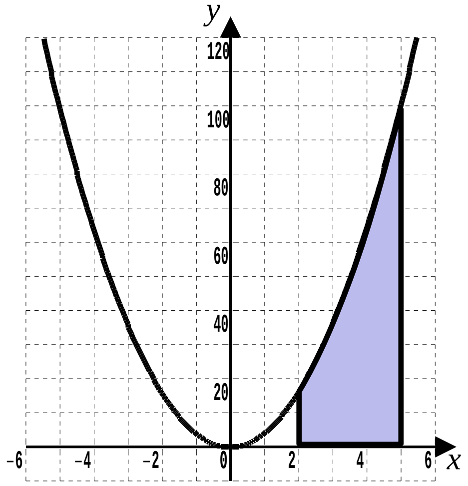


V O L U M E S of R E V O L U T I O N

Lesson 1

1.1 Introduction

Previously, integration was used to determine exactly the area under a curve.



For example, to find the area bounded by the curve $y = 4x^2$, the x-axis and the lines $x = 2$ and $x = 5$, the following piece of mathematics would be set up;

$$\begin{aligned} Area &= \int_2^5 4x^2\,dx \\ &= \left[\frac{4x^3}{3}\right]_2^5 \\ &= \left[\frac{4\times 5^3}{3}\right] - \left[\frac{4\times 2^3}{3}\right] \\ &= \left[\frac{500}{3}\right] - \left[\frac{32}{3}\right] \\ &= \left[\frac{468}{3}\right] \\ &= 156 \end{aligned}$$

Surprisingly, the mathematics to determine the volume swept out by this area as it rotates 360° about the x-axis is not much more complicated.
In an examination, the starting point is the following formula which should be memorised;

$$Volume = \pi \int y^2\,dx$$

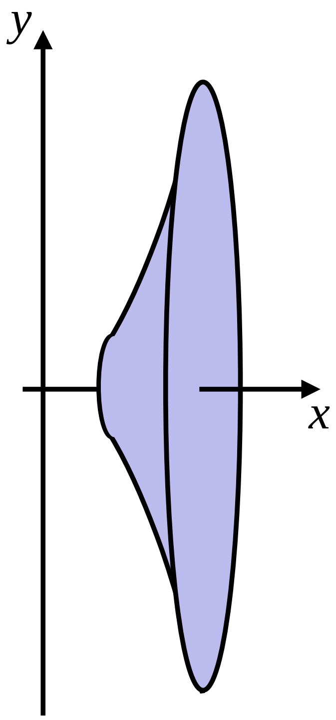
Where this comes from will be investigated shortly, but first let's master how to use it.

1.2 Example

1.2.1 The Question

Find the volume of the solid formed by rotating 360° about the x-axis the curve

$y = 4x^2$ between $x = 2$ and $x = 5$



[4 marks]

1.2.2 The Answer

$$Volume = \pi \int y^2 \, dx$$

$$= \pi \int_2^5 \left(4x^2\right)^2 dx \qquad \text{Common Error is forgetting to square the } y$$

$$= \pi \int_2^5 16x^4 \, dx$$

$$= 16\pi \int_2^5 x^4 \, dx$$

$$= 16\pi \left[\frac{x^5}{5} \right]_2^5$$

$$= 16\pi \left\{ \left[\frac{5^5}{5} \right] - \left[\frac{2^5}{5} \right] \right\}$$

$$= 16\pi \left[\frac{3093}{5} \right]$$

$$= \frac{49488\pi}{5}$$

which, if the units are cm, is about 31 000 cm^3

[4 marks]

1.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Find the volumes of the solids formed by rotating completely about the *x*-axis the areas bounded by the *x*-axis and the given curves and lines.

(i) $y = 5x^2, \quad x = -1, \quad x = 3$

[4 marks]

(ii) $y = 3x - x^2, \quad x = 1, \quad x = 2$

[4 marks]

(iii) $y = 2x - 5, \quad x = 1, \quad x = 4$

[4 marks]

(iv) $y = x^3 + 1, \quad x = -1, \quad x = 2$

[4 marks]

(v) $y = 1 + \sqrt{x}$, $x = 4$, $x = 9$

[4 marks]

(vi) $y = x + \dfrac{1}{x}$, $x = \dfrac{1}{2}$, $x = 2$

[4 marks]

Question 2

Find the volumes of the solids formed by rotating completely about the *x*-axis the areas enclosed by each of the following curves and the *x*-axis.

(i) $y = x^2 - 4$

[4 marks]

(ii) $3y = x^2 (3 - x)$

[4 marks]

Question 3

By first reflecting upon how an integration to find area can be thought of as the summation of an infinite number of infinitely thin rectangles, explain in a similar fashion where the Volume of Revolution formula comes from.

That is, why the formula to find a volume of revolution about the *x*-axis is;

$$Volume = \pi \int y^2 \, dx$$

HINT : Look up a textbook, search online...

[8 marks]