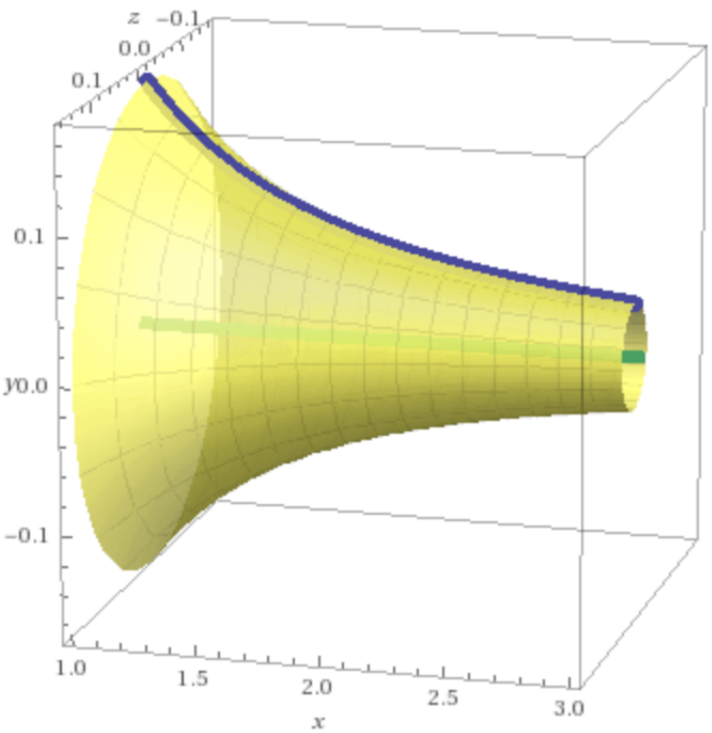
2.1 Integration Techniques

As a topic, Volumes of Revolution provides an opportunity to revise and consolidate the integration techniques taught earlier in the Year 2 A-Level course.

2.2 Partial Fractions Revisited

Find the volume of the solid generated when the profile curve $y = \dfrac{1}{\sqrt{x}\,(5x+1)}$ between $x = 1$ and $x = 3$ is rotated 2π radians about the x-axis.



[8 marks]

An answer to this question would begin by quoting the result,

$$\text{Volume} = \pi \int y^2 \, dx$$

For this particular problem,

$$\text{Volume} = \pi \int_1^3 \left(\frac{1}{\sqrt{x}\,(5x+1)} \right)^2 dx$$

$$= \pi \int_1^3 \frac{1}{x\,(5x+1)^2} \, dx$$

This tricky integration requires the use of partial fractions.

Teaching Video (Part I) : http://www.NumberWonder.co.uk/v9087/1a.mp4

Teaching Video (Part II) : http://www.NumberWonder.co.uk/v9087/1b.mp4

[8 marks]

2.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Show that the volume of the solid generated when the profile curve $y = \dfrac{1}{x\sqrt{x-1}}$ between $x = 2$ and $x = 3$ is rotated 2π radians about the x-axis is $\pi \, ln\left(\dfrac{4}{3}\right) - \dfrac{\pi}{6}$

[8 marks]

Question 2

(i) Show that $\int_1^2 \frac{1}{(4x - 3)^2}\, dx = \frac{1}{5}$

[4 marks]

(ii) Show that the volume swept out when the profile curve $y = \frac{4\sqrt{x}}{4x - 3}$

between $x = 1$ and $x = 2$ is rotated $2\pi^c$ about the x-axis is $\pi \ln 5 + \frac{12\pi}{5}$

[8 marks]

Question 3

(i) Show that $\int_1^3 \frac{1}{(2x - 1)^3}\, dx = \frac{6}{25}$

[4 marks]

(ii) Show that the volume swept out when the profile curve $y = \frac{2\sqrt{2}\, x}{(2x - 1)^{1.5}}$

between $x = 1$ and $x = 3$ is rotated $2\pi^c$ about the x-axis is given by,

$$Volume = \pi \ln 5 + \frac{52\pi}{25}$$

[8 marks]

Question 4

Find the volume of the solid generated when the profile curve $y = \dfrac{2}{\sqrt{x}\ (3x - 2)}$

between $x = 1$ and $x = 2$ is rotated 2π radians about the x-axis.

[8 marks]