Lesson 2

GCSE and Preparatory A-Level Mathematics

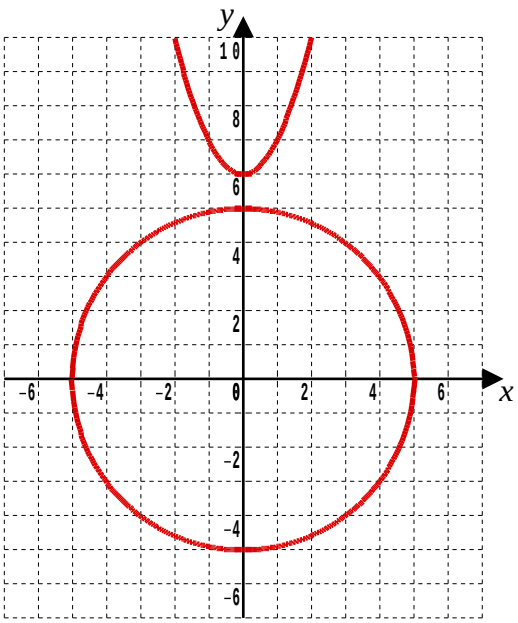
Conic Sections (Simultaneous Equations IV)

2.1 Circle and Parabola

The circle with equation $x^2 + y^2 = 25$ has centre (0, 0) and radius 5 units.

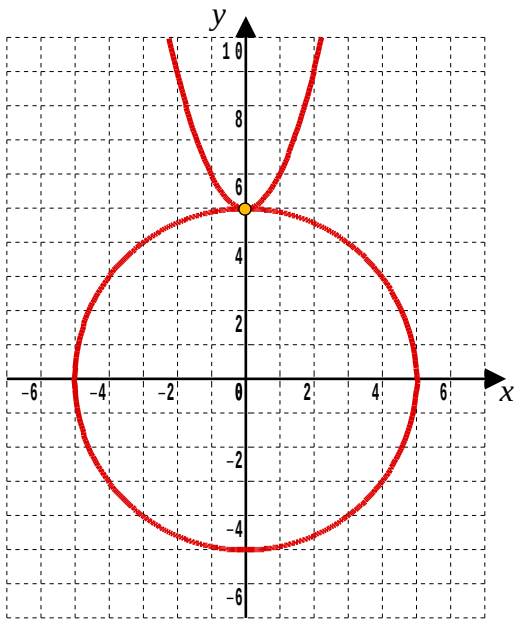
Into this circle is going to be lowered a parabola, essentially $y = x^2$

The lowering is achieved by varying the value of c in the equation $y = x^2 + c$



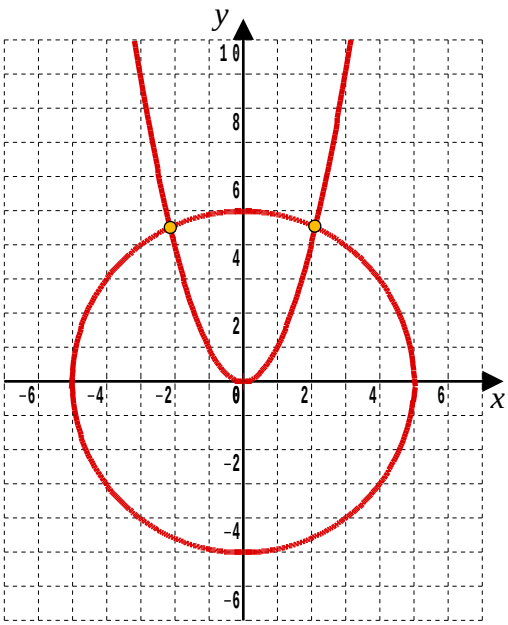
$x^2 + y^2 = 25$ with $y = x^2 + 6$

Zero solutions



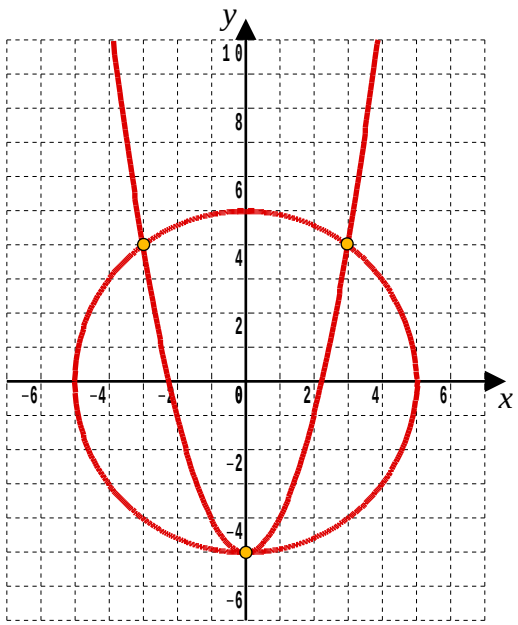
$x^2 + y^2 = 25$ with $y = x^2 + 5$

One solution



$x^2 + y^2 = 25$ with $y = x^2$

Two solutions



$x^2 + y^2 = 25$ with $y = x^2 - 5$

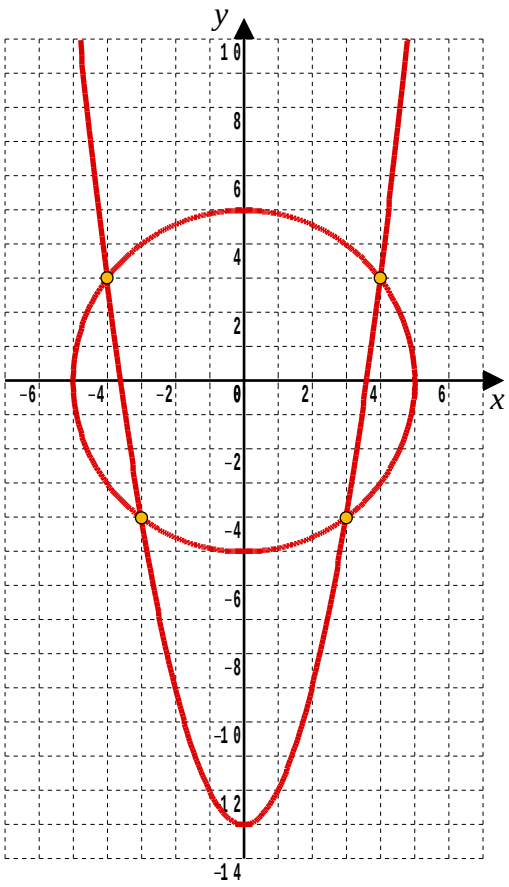
Three solutions

The fifth and final possibility is that there are four solutions.

$$x^2 + y^2 = 25 \quad \text{with } y = x^2 - 13$$

Geometrically, we can see from a graph that there are four solutions. The challenge now is to use algebra to find all four solutions.

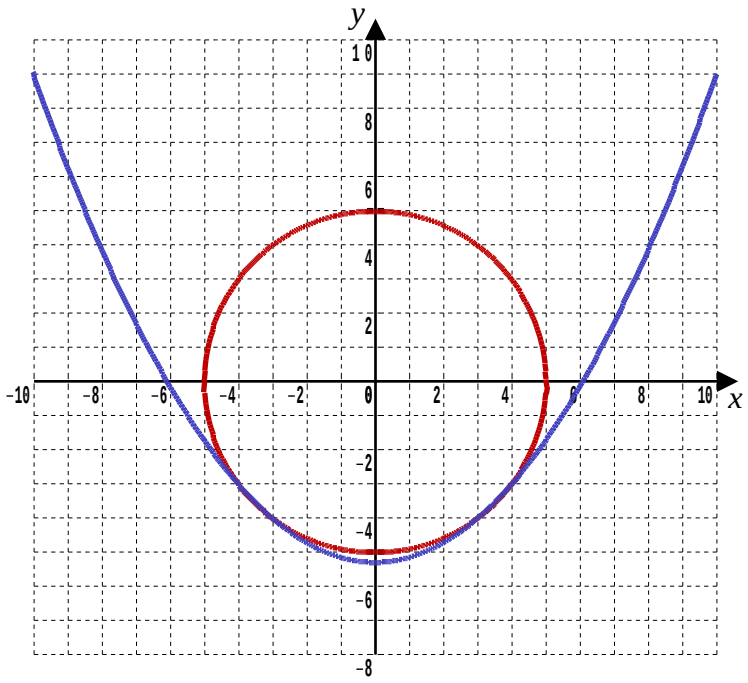
Teaching Videos : http://www.NumberWonder.co.uk/v9091/2.mp4



2.2 Exercise

Question 1

The number of intersection points is often not obvious from a graph !

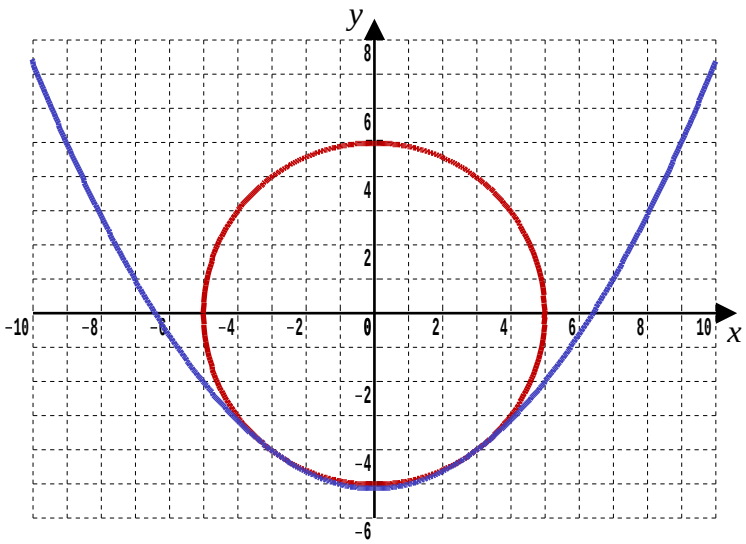


Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$

with the parabolic curve, $y = \dfrac{x^2 - 37}{7}$

Question 2

Sophia is trying to get more than four points of intersection by running the quadratic curve along an arc of the circle. Here is her first attempt.

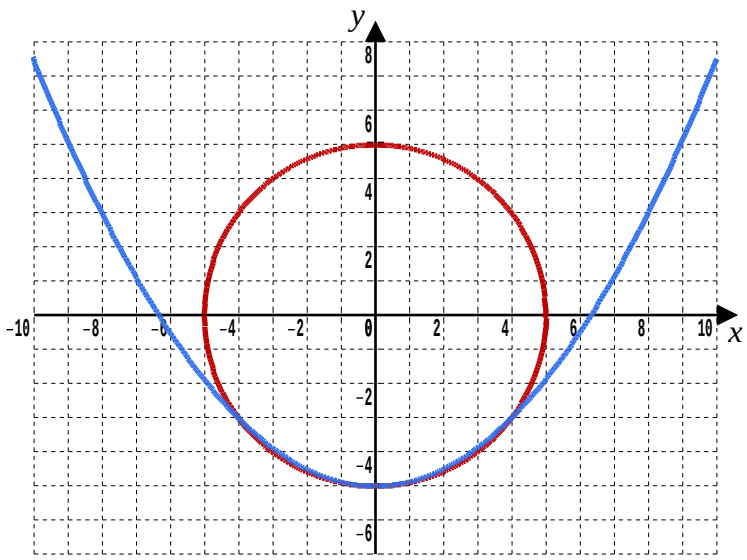


Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$

with the quadratic curve, $y = \dfrac{x^2 - 41}{8}$

Question 3

Sophia is does not give up easily. Here is her next attempt.

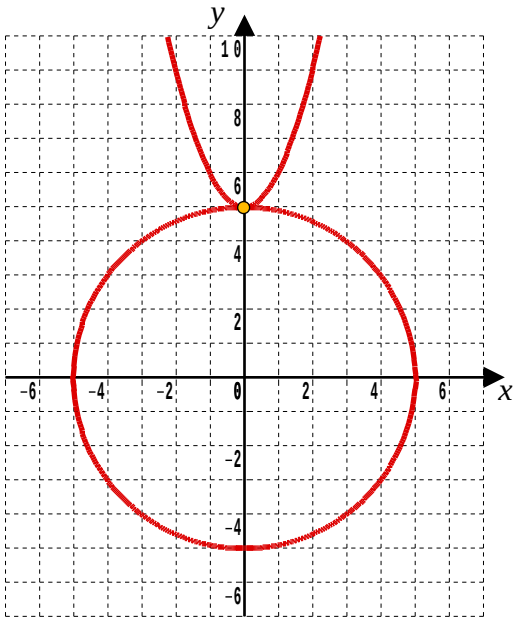


(i) Use algebra to find all points of intersection of the circle $x^2 + y^2 = 25$ with the quadratic curve, $y = \dfrac{x^2 - 40}{8}$

(ii) Do you think Sophia's idea can be made to work ?
Yes or No ?

Question 4

In the introduction it was claimed that the circle $x^2 + y^2 = 25$ would only have one point of intersection with the parabola $y = x^2 + 5$



Verify this claim by using algebra to solve the two equations simultaneously.