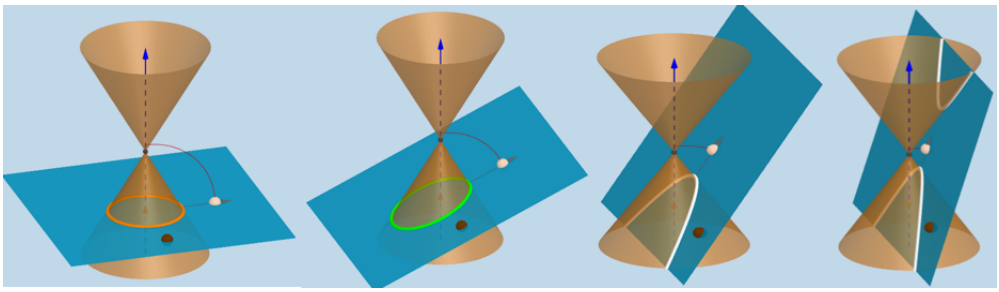
Lesson 3

GCSE and Preparatory A-Level Mathematics

Conic Sections (Simultaneous Equations IV)

3.1 Circle and Hyperbola

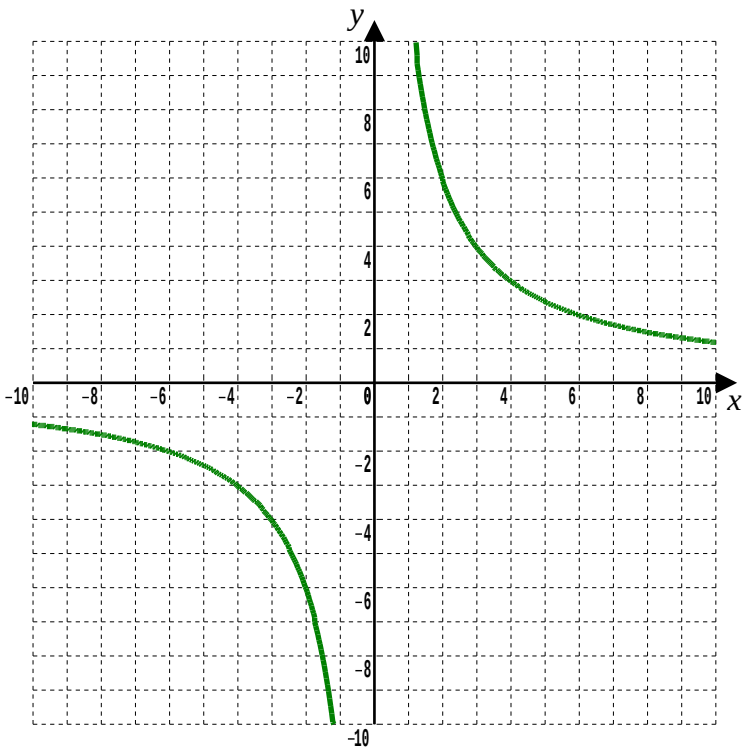
The circle, ellipse, parabola and the hyperbola form a category of curves known collectively as "Conic Sections". This name comes from the fact that they can all occur when a double cone (each of infinite extent) is sliced in various ways.



circle ellipse parabola hyperbola

The graph of a hyperbola is shown below. In this form it's known as the graph of inverse proportion or, more informally, as the "more is less" graph.

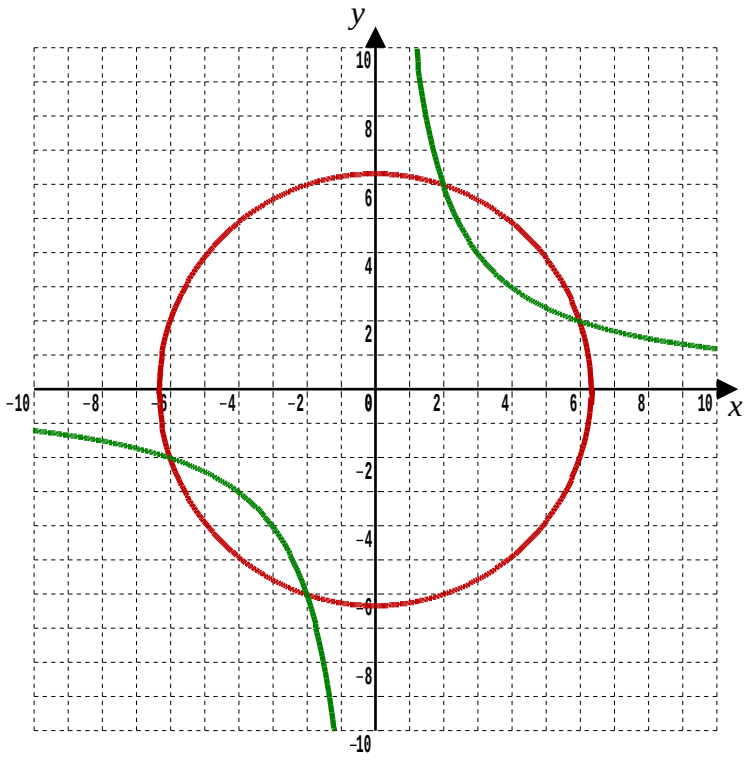
This particular hyperbola has the equation $y = \frac{12}{x}$



For GCSE and A-Level the hyperbolas considered are all of the form $y = \frac{k}{x}$

where k is a constant. In the graph above that constant is 12.

3.2 Quadratic In Disguise



Use algebra to find the points where $x^2 + y^2 = 40$ and $y = \frac{12}{x}$ intersect.

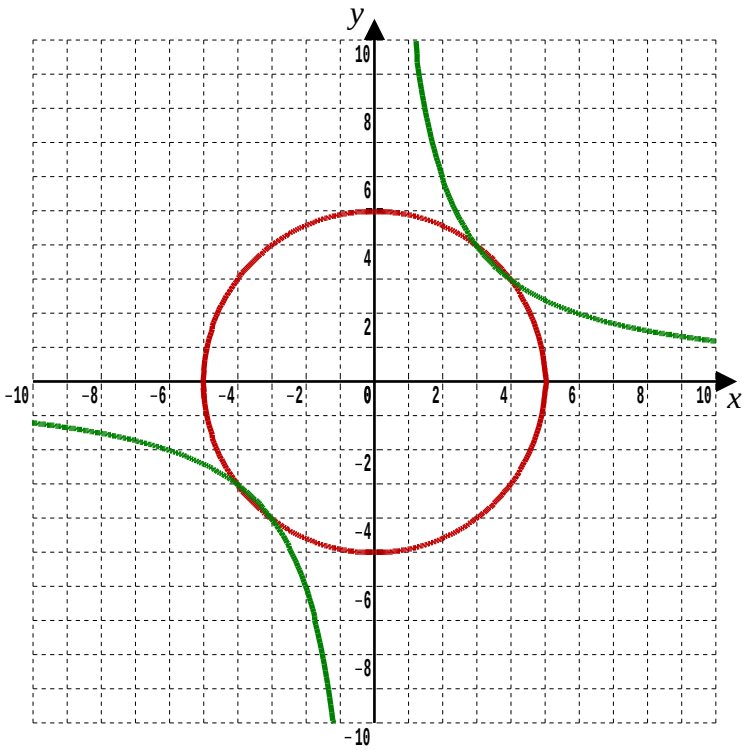
Teaching Video : http://www.NumberWonder.co.uk/v9091/3a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9091/3b.mp4 (Part 2)

⇐ Part 1 Part 2 ⇒

3.3 Exercise

Question 1

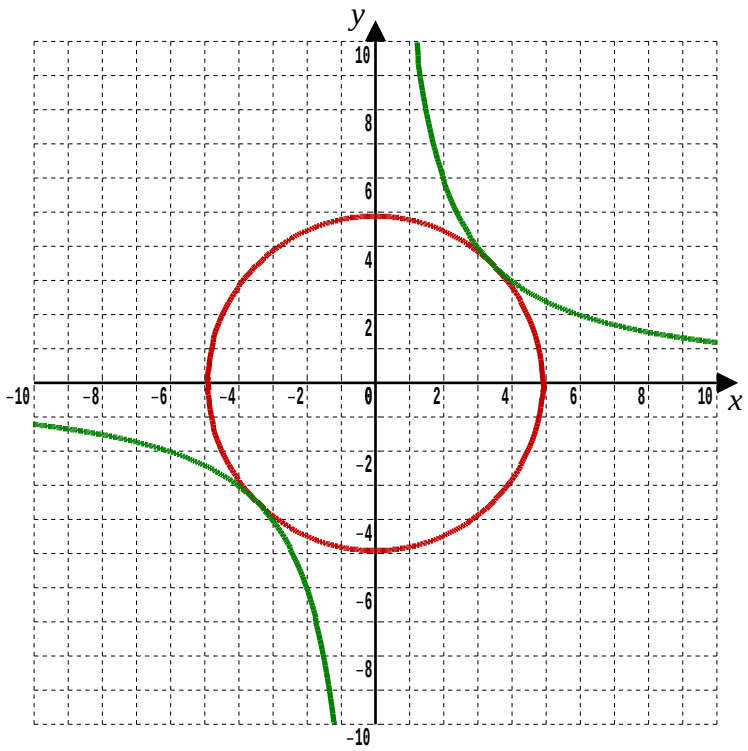
Fran has decided to shrink the circle to have a radius of 5.



Use algebra to find the points where $x^2 + y^2 = 25$ and $y = \frac{12}{x}$ intersect.

Question 2

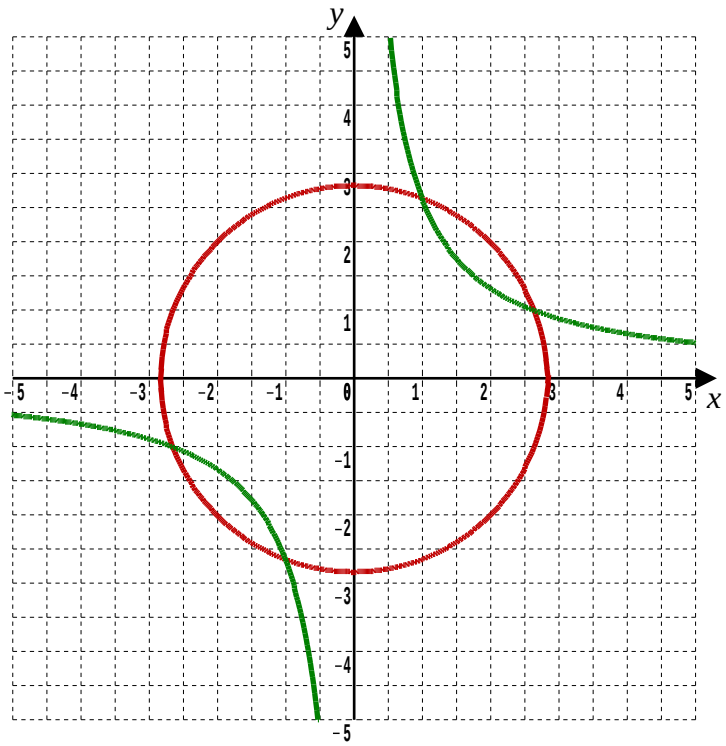
Oscar wonders if he can get a hyperbola to touch a circle in only two points. After playing around with a graph plotter, he arrives at the following graph.



Use algebra to find the points where $x^2 + y^2 = 18$ and $y = \frac{9}{x}$ intersect and so find out if Oscar has succeeded in his mission.

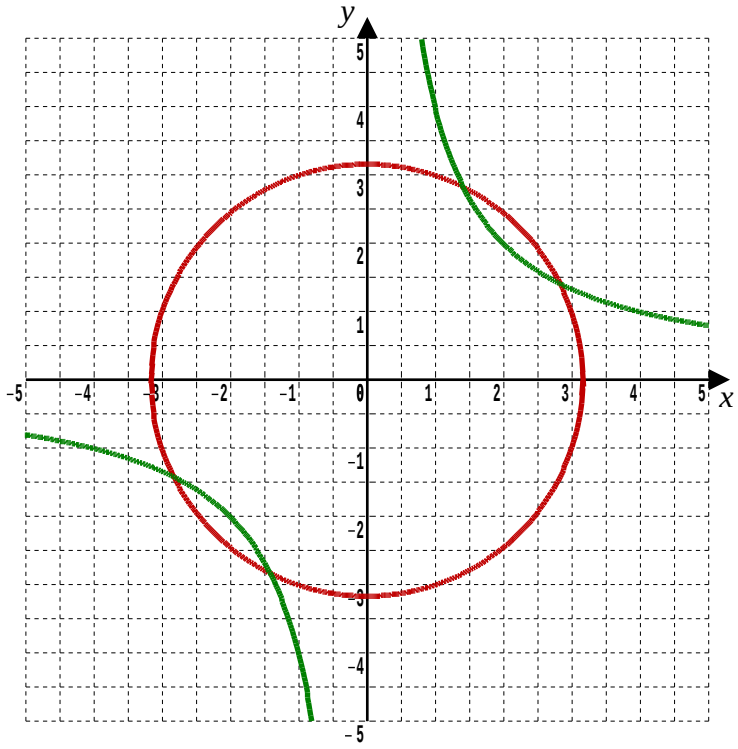
Question 3

In this question the intersections are not points with all integer coordinates.
One part of each point involves the square roots of an integer.
The graph is still useful to check that answers obtained are not obviously wrong !



Use algebra to find the points where $x^2 + y^2 = 8$ and $y = \frac{\sqrt{7}}{x}$ intersect.

Question 4



Use algebra to find the points where $x^2 + y^2 = 10$ and $y = \frac{4}{x}$ intersect