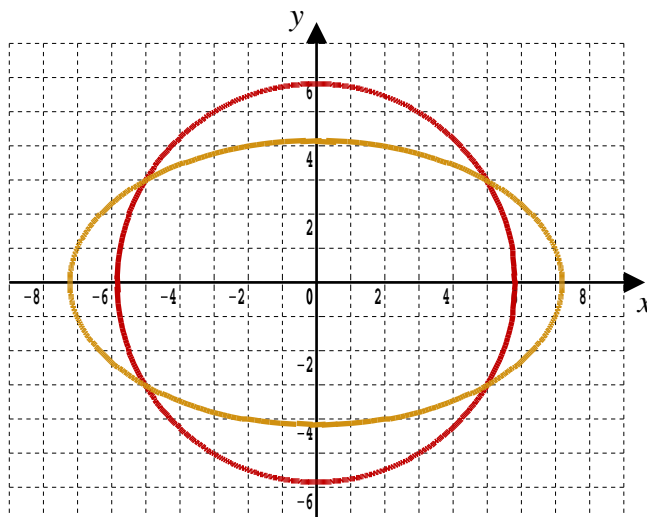


Lesson 4

GCSE and Preparatory A-Level Mathematics Conic Sections (Simultaneous Equations IV)

4.1 Circle and Ellipse

In appearance, a circle (in red) and an ellipse (in gold) have much in common. They are both smooth and continuous curves that form a graceful closed loop.



No surprise then that their algebraic equations also look similar.

$$\text{Red circle : } x^2 + y^2 = 34$$

$$\text{Gold ellipse : } x^2 + 3y^2 = 52$$

More generally,

The Equation of a Circle

$$x^2 + y^2 = r^2$$

This is a circle with centre (0, 0) and radius r

The Equation of an Ellipse

$$x^2 + ay^2 = w^2$$

This is an ellipse with centre (0, 0) and *half width*, w

In fact, when $a = 1$ in the equation of an ellipse, it becomes the equation of a circle.

This is because, $a = \left(\frac{\text{half width}}{\text{half height}} \right)^2$

and for a circle the *half width* and the *half height* are the same; the *radius*, r .

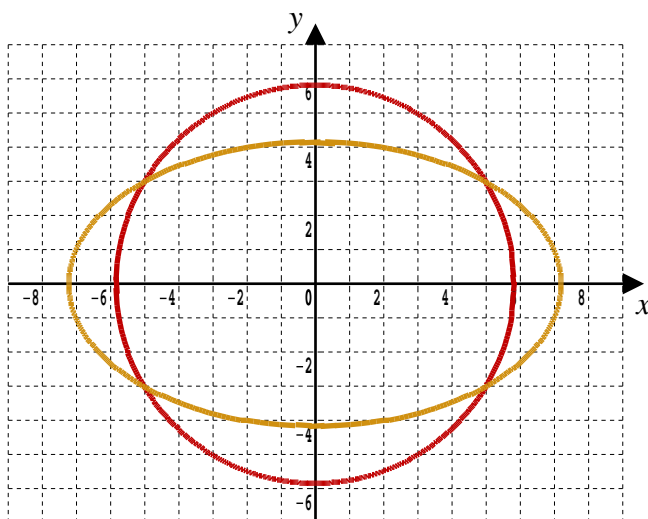
- The *half width* is the number where the ellipse crosses the positive x -axis.
- The *half height* is the number where the ellipse crosses the positive y -axis.

4.2 Where Gold meets Red

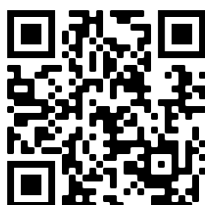
Use algebra to solve the simultaneous equations,

$$\text{Red circle : } x^2 + y^2 = 34$$

$$\text{Gold ellipse : } x^2 + 3y^2 = 52$$



Teaching Video : <http://www.NumberWonder.co.uk/v9091/4.mp4>



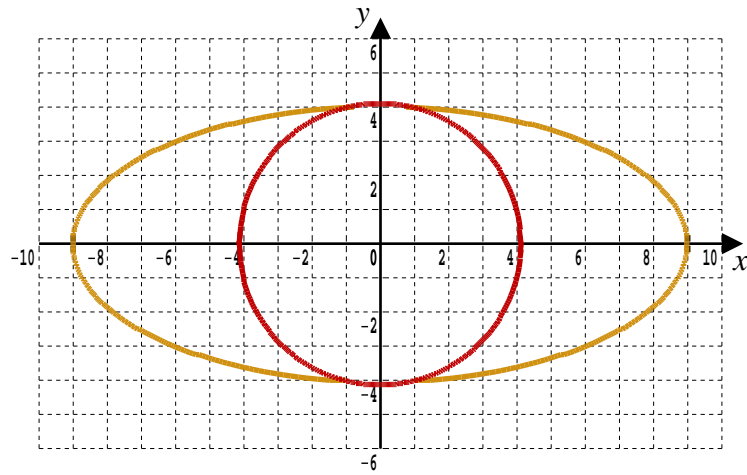
4.3 Exercise

Question 1

Use algebra to solve the simultaneous equations,

$$\text{Red circle : } x^2 + y^2 = 17$$

$$\text{Gold ellipse : } x^2 + 5y^2 = 81$$

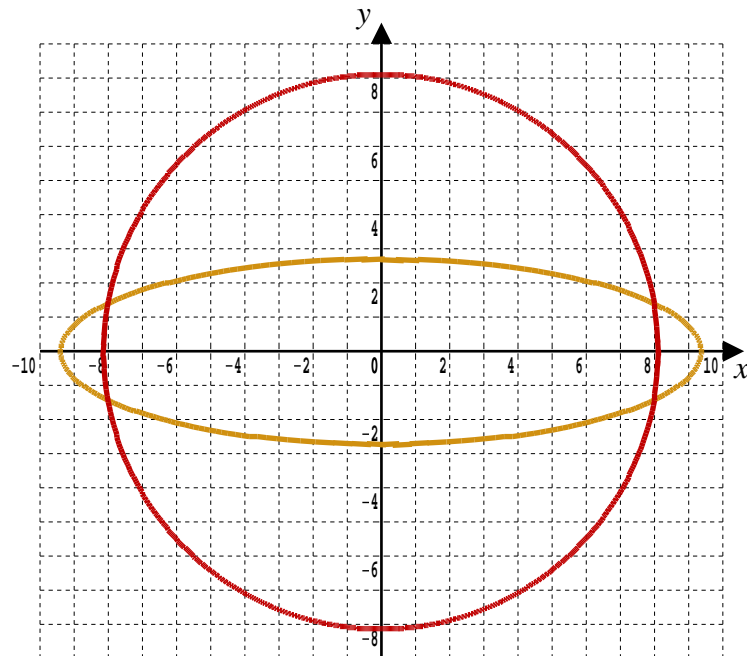


Question 2

Use algebra to solve the simultaneous equations,

$$\text{Red circle : } x^2 + y^2 = 66$$

$$\text{Gold ellipse : } x^2 + 12y^2 = 88$$



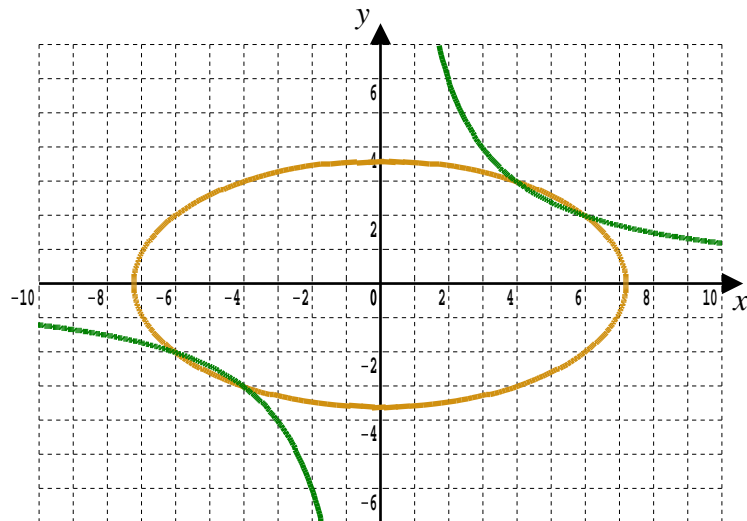
Give your points as exact coordinates, leaving square roots in your answers.

Question 3

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse : } x^2 + 4y^2 = 52$$

$$\text{Green hyperbola : } y = \frac{12}{x}$$



This list of pairs of positive integers that have a product of 576 may be of use !

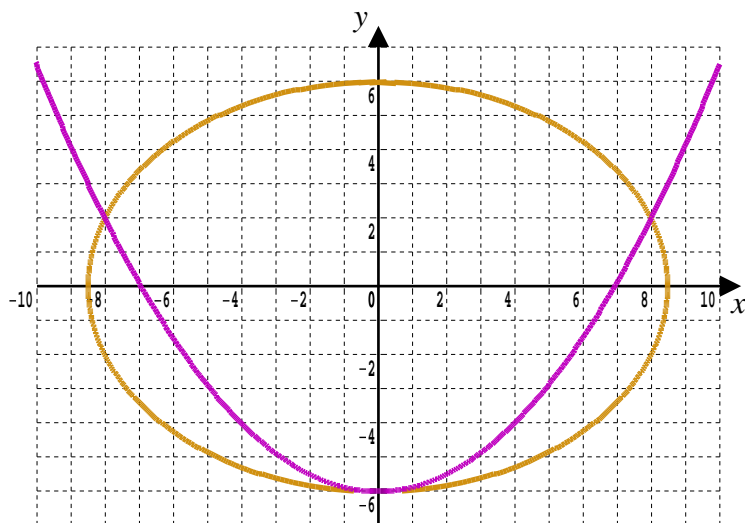
- 1×576
- 2×288
- 3×192
- 4×144
- 6×96
- 8×72
- 9×64
- 12×48
- 16×36
- 18×32
- 24×24

Question 4

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse : } x^2 + 2y^2 = 72$$

$$\text{Purple parabola : } y = \frac{x^2}{8} - 6$$



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In October 2020, Shrewsbury School was voted "**Independent School of the Year 2020**"

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com