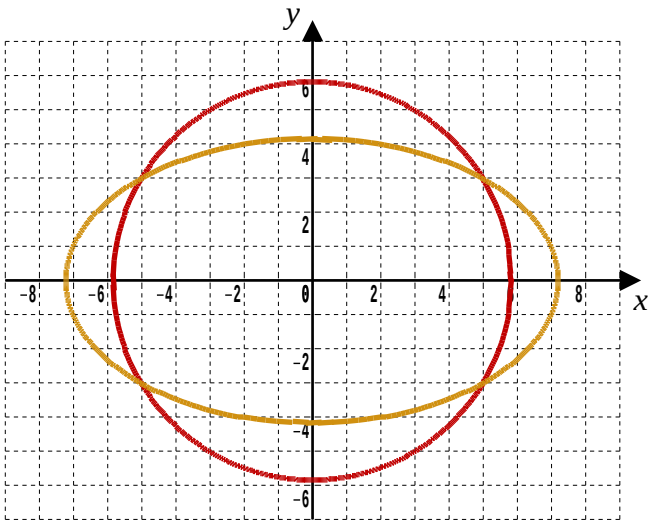
Lesson 4

GCSE and Preparatory A-Level Mathematics

Conic Sections (Simultaneous Equations IV)

4.1 Circle and Ellipse

In appearance, a circle (in red) and an ellipse (in gold) have much in common. They are both smooth and continuous curves that form a graceful closed loop.



No surprise then that their algebraic equations also look similar.

$$\text{Red circle :} \quad x^2 + y^2 = 34$$

$$\text{Gold ellipse :} \quad x^2 + 3y^2 = 52$$

More generally,

The Equation of a Circle

$$x^2 + y^2 = r^2$$

This is a circle with centre (0, 0) and radius r

The Equation of an Ellipse

$$x^2 + a\,y^2 = w^2$$

This is an ellipse with centre (0, 0) and *half width*, w

In fact, when $a = 1$ in the equation of an ellipse, it becomes the equation of a circle.

This is because, $\quad a = \left(\dfrac{half\ width}{half\ height} \right)^2$

and for a circle the *half width* and the *half height* are the same; the *radius*, r.

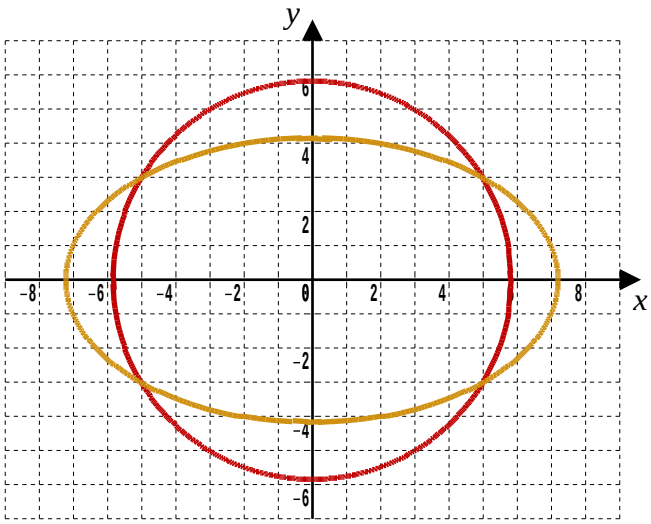
- The *half width* is the number where the ellipse crosses the positive x-axis.
- The *half height* is the number where the ellipse crosses the positive y-axis.

4.2 Where Gold meets Red

Use algebra to solve the simultaneous equations,

$$\text{Red circle : } \quad x^2 + y^2 = 34$$

$$\text{Gold ellipse : } \quad x^2 + 3y^2 = 52$$



Teaching Video : http://www.NumberWonder.co.uk/v9091/4.mp4

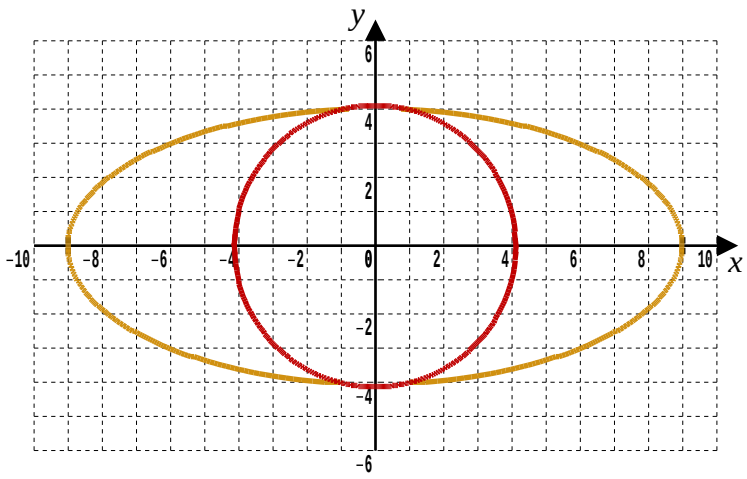
4.3 Exercise

Question 1

Use algebra to solve the simultaneous equations,

$$\text{Red circle}: \quad x^2 + y^2 = 17$$

$$\text{Gold ellipse}: \quad x^2 + 5y^2 = 81$$

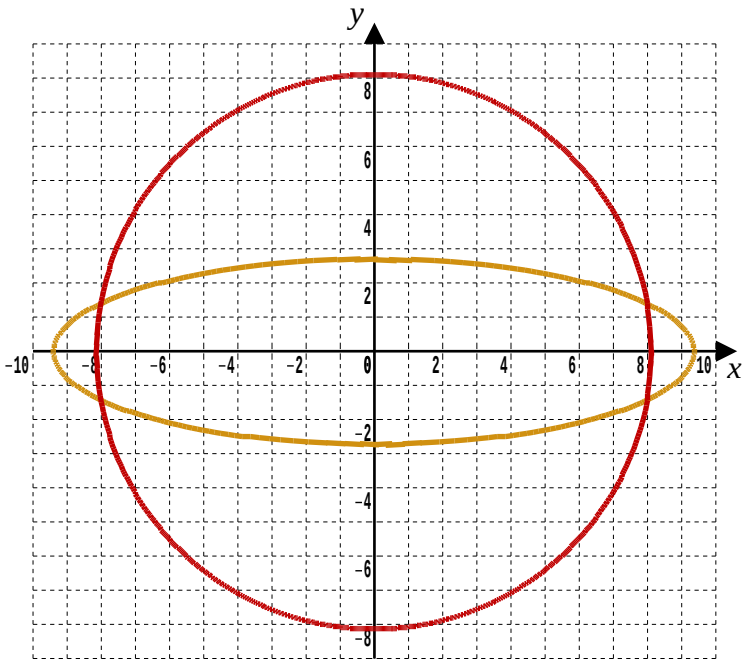


Question 2

Use algebra to solve the simultaneous equations,

$$\text{Red circle}: \quad x^2 + y^2 = 66$$

$$\text{Gold ellipse}: \quad x^2 + 12\,y^2 = 88$$



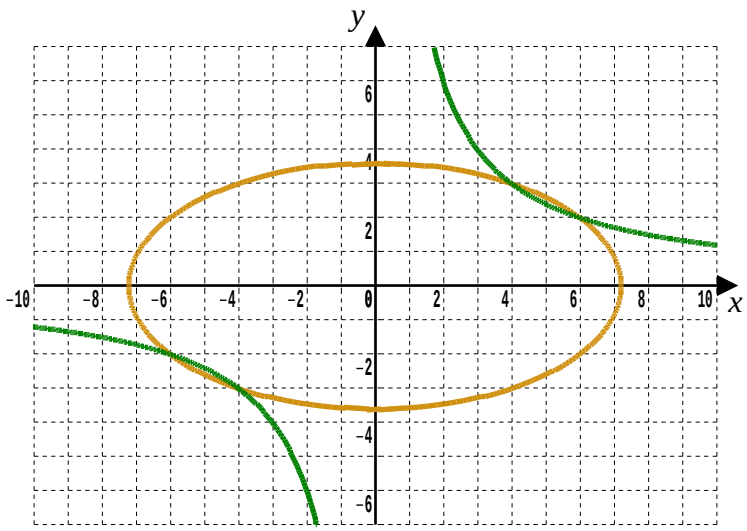
Give your points as exact coordinates, leaving square roots in your answers.

Question 3

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse : } \quad x^2 + 4y^2 = 52$$

$$\text{Green hyperbola : } \quad y = \frac{12}{x}$$



This list of pairs of positive integers that have a product of 576 may be of use !

1×576
2×288
3×192
4×144
6×96
8×72
9×64
12×48
16×36
18×32
24×24

Question 4

Use algebra to solve the simultaneous equations,

$$\text{Gold ellipse : } \quad x^2 + 2y^2 = 72$$

$$\text{Purple parabola : } \quad y = \frac{x^2}{8} - 6$$