Conic Sections (Simultaneous Equations IV)

5.1 The Q Formula

In examinations candidates are allowed to use the Q Formula;

(The Q stands for Quadratic)

At GCSE level, it is given on a page along with several other useful formulae.

For A-Level candidates are expected to have memorised it.

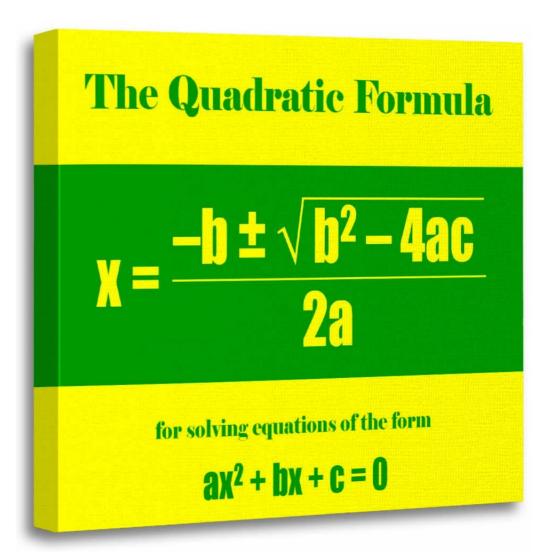
The Q Formula

A quadratic equation that is written in the form

$$ax^2 + bx + c = 0$$
 where a, b and c are constants

has real solutions, if any exist, given by the formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

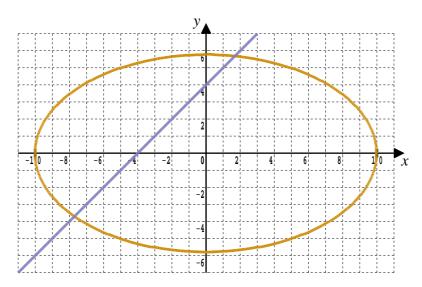


5.2 Q Formula In Use

Use algebra to solve the simultaneous equations,

Purple line : y = x + 4

Gold ellipse: $x^2 + 3y^2 = 100$



Teaching Video: http://www.NumberWonder.co.uk/v9091/5a.mp4 (Part 1)
http://www.NumberWonder.co.uk/v9091/5b.mp4 (Part 2)



<= Part 1

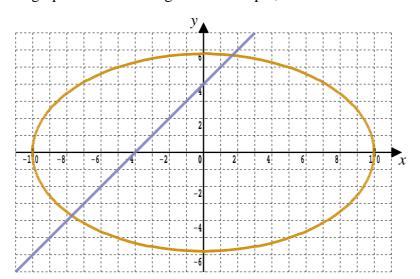
Part 2 =>



5.3 Exercise

Question 1

Here is the graph of the Teaching Video example, once more.



In the video the exact coordinates of the points of intersection were found to be,

$$(-3 - \sqrt{22}, 1 - \sqrt{22})$$
 $(-3 + \sqrt{22}, 1 + \sqrt{22})$

Using a calculator, write these exact answers as decimal approximations accurate to one decimal place.

Do your decimal coordinates approximately match up with where the graph shows that the straight line and the ellipse cross ?

Question 2

Showing working, simplify the following numbers.

Your answers should all be of the form $p \pm \sqrt{q}$ where p and q are integers.

$$(\mathbf{i}) \qquad \frac{9 \pm \sqrt{18}}{3}$$

(ii)
$$\frac{35 \pm \sqrt{50}}{5}$$

$$(\mathbf{iii}) \quad \frac{56 \pm \sqrt{48}}{4}$$

$$(iv)$$
 $\frac{-24 \pm \sqrt{640}}{8}$

Question 3

Showing working, simplify the following numbers.

Your answers should all be of the form $r \pm s\sqrt{t}$ where r, s and t are integers.

(i)
$$\frac{10 \pm \sqrt{300}}{5}$$

(ii)
$$\frac{-45 \pm \sqrt{360}}{3}$$

Question 4

Show how to use the Q Formula to solve the following quadratic equations, Your answers should all be of the form $p \pm \sqrt{q}$ where p and q are integers.

(i)
$$x^2 + 8x + 5 = 0$$

(ii)
$$x^2 + 2x - 9 = 0$$

Question 5

Use algebra to solve the simultaneous equations,

Purple line :
$$y = x + 7$$

Gold ellipse:
$$x^2 + 6y^2 = 84$$

Write your answers as simplified exact points.

