Lesson 4

Further A-Level Pure Mathematics : Core 1

Roots of Polynomials

4.1 Root Relations II

As with quadratics, with cubics it may be that the equation of a second cubic is sought that is related to the first through a transformation of the roots. Two methods of tackling such questions will be presented. In the first it is the roots that are worked on, in the second the equation is transformed directly.

4.2 Example

The roots of the cubic equation $3x^3 + x^2 + 2x + 6 = 0$ are α, β and γ.

Without solving the equation, find an all integer coefficient polynomial with roots

$$3\alpha, \; 3\beta \; \text{ and } \; 3\gamma$$

Teaching Video : http://www.NumberWonder.co.uk/v9093/4a.mp4 (Method 1)
http://www.NumberWonder.co.uk/v9093/4b.mp4 (Method 2)

<=Method 1 Method 2 =>

After watching the teaching videos write out your own version of the solution.

☞

[6 marks]

Both methods are useful. For some questions either method will work, for others only one of the methods will. If one method stalls, try the other.

4.3 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable.
Make the method used clear.

Marks available : 40

Question 1

The roots of the cubic equation $x^3 + mx^2 + nx + 5 = 0$ are α, β and γ.

(i) Given that $\alpha + \beta + \gamma = 7$ and $\alpha\beta + \beta\gamma + \gamma\alpha = 6$ state the value of m and the value of n.

[2 marks]

(ii) Without solving the equation, find an all integer coefficient polynomial with roots 2α, 2β and 2γ.

[3 marks]

Question 2

The roots of the cubic equation $x^3 + 10x^2 + 3x - 7 = 0$ are α, β and γ.
Without solving the equation, find an all integer coefficient polynomial with roots

$$\alpha - 1, \ \beta - 1 \ \text{ and } \ \gamma - 1$$

[5 marks]

Question 3

Further A-Level Examination Question from May 2018, Core 1, Q2 (Edexcel)

The cubic equation $z^3 - 3z^2 + z + 5 = 0$ has roots α, β and γ

Without solving the equation, find the cubic equation whose roots are $(2\alpha + 1)$, $(2\beta + 1)$ and $(2\gamma + 1)$ giving your answer in the form $w^3 + pw^2 + qw + r$ where p, q and r are integers to be found.

[5 marks]

Question 4

The cubic equation $x^3 + 5x^2 - x + 3 = 0$ has roots α, β and γ

Without solving the equation, find the cubic equation whose roots are α^2, β^2 and γ^2 giving your answer in the form $w^3 + pw^2 + qw + r$ where p, q and r are integers.

[6 marks]

Question 5

The cubic equation $x^3 + 4x^2 - 5x - 7 = 0$ has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are,

$$\frac{1}{\alpha + 1}, \qquad \frac{1}{\beta + 1} \quad \text{and} \quad \frac{1}{\gamma + 1}$$

giving your answer in the form $w^3 + pw^2 + qw + r$ where p, q and r are integers.

[6 marks]

Question 6

A quintic polynomial has equation,

$$x^5 + x + 2 = 0$$

and its roots are $\alpha, \beta, \gamma, \delta, \sigma$

Without solving the equation, find the quintic with roots,

$$\alpha + 2,\ \beta + 2,\ \gamma + 2,\ \delta + 2 \text{ and } \sigma + 2$$

[5 marks]

Question 7

A cubic polynomial with roots α, β and γ has the equation,

$$x^3 + 2x^2 - 3x - 5 = 0$$

Without solving the equation, find an equation that has roots

$$\alpha + \beta,\ \beta + \gamma \text{ and } \gamma + \alpha$$

[8 marks]