

5.1 From the Examination Hall**Summary**

Proof by induction is used to prove that a statement is true for all positive integers.

A typical proof by induction will have four main steps;

- Basis The statement is proved true for $n = 1$
- Assumption The statement is assumed to be true for $n = k$
- Inductive The statement is shown, in consequence, to be true for $n = k + 1$
- Conclusion The statement is consequently deduced true for all positive integers n

Cases Considered

- Proving divisibility by an integer
- Proving a position-to-term formula for a sequence defined by a term to term formula
- Proving a relationship about a matrix raised to the power n
- Proving a formula for the sum of a series, often involving sigma notation

As with all proofs, what examiners are looking for is the quality of the logical reasoning and an attention to the detail.

5.2 Exercise

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 33

Question 1

Further AS-Level Sample Assessment Materials, Core 1, Q6 (a) (Edexcel)

Prove by induction that for all positive integers n ,

$$\sum_{r=1}^n r^2 = \frac{1}{6} n (n + 1)(2n + 1)$$

[6 marks]

Question 2

Further A-Level Sample Assessment Materials, Core 1, Q2 (Edexcel)

Prove by induction that for all positive integers n

$$f(n) = 2^{3n+1} + 3(5^{2n+1})$$

is divisible by 17

[6 marks]

Question 3*Further A-Level Examination Question from May 2016, Q8 (Edexcel)*

(i) Prove by induction that, for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^n \frac{2r + 1}{r^2(r + 1)^2} = 1 - \frac{1}{(n + 1)^2}$$

[5 marks]

(ii) A sequence of positive rational numbers is defined by,

$$u_1 = 3$$

$$u_{n+1} = \frac{1}{3} u_n + \frac{8}{9} \quad n \in \mathbb{Z}^+$$

Prove by induction that, for $n \in \mathbb{Z}^+$

$$u_n = 5 \times \left(\frac{1}{3} \right)^n + \frac{4}{3}$$

[5 marks]

Question 4

Further A-Level Examination Question from May 2018, IAL F1, Q8 (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} a & 0 \\ 1 & b \end{pmatrix}^n = \begin{pmatrix} a^n & 0 \\ \frac{a^n - b^n}{a - b} & b^n \end{pmatrix}$$

where a and b are constants and $a \neq b$

[5 marks]

Question 5

Further A-Level Examination Question, January 2017, IAL, F1, Q9 (ii) (Edexcel)

Prove by induction that, for $n \in \mathbb{Z}^+$

$$f(n) = 5^{2n} + 3n - 1$$

is divisible by 9

[6 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk