2.1 Streamlining

If the process of finding the best polynomial of degree n to a given function is streamlined, the resulting general rule is termed the Maclaurin Series. It's traditionally written in function notation rather than Leibniz notation.

The Maclaurin Series

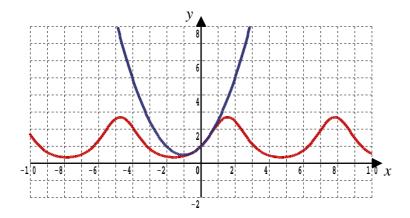
A given function, f(x), may be written as the polynomial,

$$f(x) = f(0) + f'(0) x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(r)}(0)}{r!} x^r + \dots$$

provided that f(0), f'(0), f''(0), ..., $f^{(r)}(0)$, ... all have finite values.

2.2 Example

Find the first three terms of the Maclaurin series for $f(x) = e^{\sin x}$



Teaching Video: http://www.NumberWonder.co.uk/v9098/2.mp4



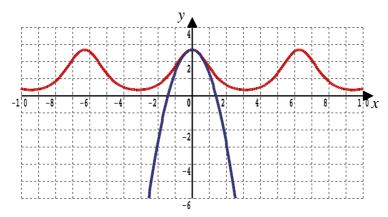
2.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 50

Question 1

Find the first three terms of the Maclaurin series for $f(x) = e^{\cos x}$

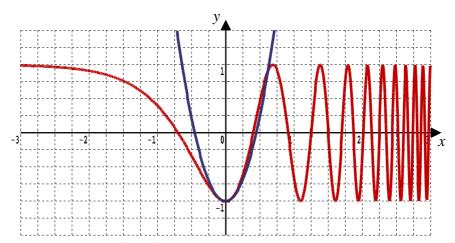


In red is the graph of $y = e^{\cos x}$ and in blue the best quadratic approximation centred on x = 0

[6 marks]

Question 2

Find the first three terms of the Maclaurin series for $f(x) = cos(\pi e^x)$

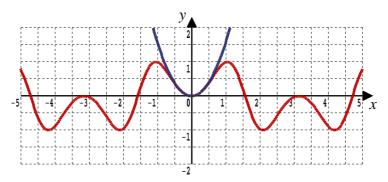


In red is the graph of $y = cos(\pi e^x)$ and in blue the best quadratic approximation centred on x = 0

[6 marks]

Question 3

Find the first three terms of the Maclaurin series for $f(x) = sin(\pi cos x)$

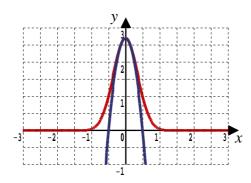


In red is the graph of $y = sin(\pi cos x)$ and in blue the best quadratic approximation centred on x = 0

[6 marks]

Question 4

Find the first three terms of the Maclaurin series for $f(x) = e^{1-x^2}$



In red is the graph of $y = e^{1-x^2}$ and in blue the best quadratic approximation centred on x = 0

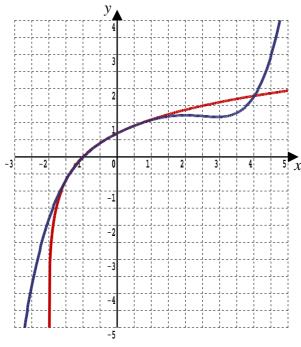
[6 marks]

Question 5

(i) Complete the table to show the first, second, third, fourth and fifth derivatives of the function y = ln(2 + x) where x > -2 Also work out values of those derivatives at x = 0

$f(x) = \ln(2 + x)$	when $x = 0$,	f(0) =
f'(x) =	when $x = 0$,	f'(0) =
f''(x) =	when $x = 0$,	f"(0) =
f'''(x) =	when $x = 0$,	f'''(0) =
$f^{(4)}(x) =$	when $x = 0$,	$f^{(4)}(0) =$
$f^{(5)}(x) =$	when $x = 0$,	$f^{(5)}(0) =$

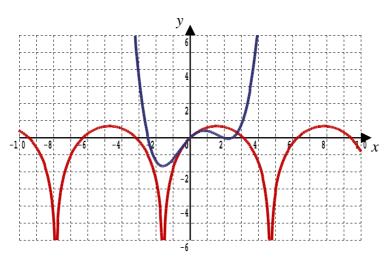
(ii) Hence determine the best quintic polynomial approximation to the function y = ln(2 + x) for values of x that are close to zero.



In red is the graph of y = ln(2 + x) and in blue the best quintic polynomial approximation on x = 0

Question 6

Showing full working, determine the best quartic polynomial approximation to the function y = ln(1 + sin x) for values of x that are close to zero.



In red is the graph of y = ln(1 + sin x) and in blue the best quartic polynomial approximation on x = 0

[10 marks]

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 $Teachers\ may\ obtain\ detailed\ worked\ solutions\ to\ the\ exercises\ by\ email\ from\ mhh@shrewsbury.org.uk$