### 4.1 The Inverse Trig Functions

A few derivatives of inverse trigonometric functions are amongst those given in the examination formula booklet. Even so, an examination may well ask for one of the results given to be proven.

f(x)	f'(x)	In Formula Book?
$x^n$	$n x^{n-1}$	No
$e^x$	$e^x$	No
ln x	$\frac{1}{x}$	No
sin x	cos x	No
cos x	$-\sin x$	No
tan x	$sec^2 x$	Yes
csc x	$-\csc x \cot x$	Yes
sec x	sec x tan x	Yes
cot x	$-csc^2x$	Yes
arcsin x	$\frac{1}{\sqrt{1-x^2}}$	Yes
arccos x	$-\frac{1}{\sqrt{1-x^2}}$	Yes
arctan x	$\frac{1}{1+x^2}$	Yes

# Example #1

Prove that the derivative of  $\arcsin x$  is  $\frac{1}{\sqrt{1-x^2}}$ 

Teaching Video: <a href="http://www.NumberWonder.co.uk/v9028/10b.mp4">http://www.NumberWonder.co.uk/v9028/10b.mp4</a>



### 4.2 The Binomial Theorem

To find the Maclaurin series for arcsin x use could be made of the rule,

### The Maclaurin Series

A given function, f(x), may be written as the polynomial,

$$f(x) = f(0) + f'(0) x + \frac{f''(0)}{2!} x^2 + \dots + \frac{f^{(r)}(0)}{r!} x^r + \dots$$

provided that f(0), f'(0), f''(0), ...,  $f^{(r)}(0)$ , ... all have finite values.

However, the Binomial Theorem provides an easier way.

## Example #2

With the help of the Binomial Theorem, find the first four non-zero terms in the Maclaurin series for  $arcsin\ x$ 

Teaching Video: http://www.NumberWonder.co.uk/v9098/4.mp4



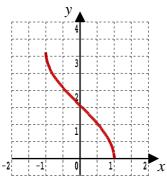
#### 4.3 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 40

# **Question 1**

With the aid of the graph, prove that the derivative of  $\arccos x$  is  $-\frac{1}{\sqrt{1-x^2}}$ 



y = arccos x

[ 5 marks ]

# **Question 2**

(i) It is obviously true that, 
$$\frac{1}{\sqrt{1-x^2}} + \left(-\frac{1}{\sqrt{1-x^2}}\right) = 0$$

By integrating each term in this equation, deduce a simple relationship between  $\arcsin x$  and  $\arccos x$ 

[ 3 marks ]

(ii) Hence, use the Maclaurin series for arcsin x to deduce the Maclaurin series for arccos x.

[ 2 marks ]

# **Question 3**

Prove that the derivative of  $\arctan x$  is  $\frac{1}{1+x^2}$ 

[ 5 marks ]

# **Question 4**

With the help of the Binomial Theorem, find the first four non-zero terms in the Maclaurin series for arctan x

# **Question 5**

Prove that the derivative of arccot x is  $-\frac{1}{1+x^2}$ 

[ 5 marks ]

# **Question 6**

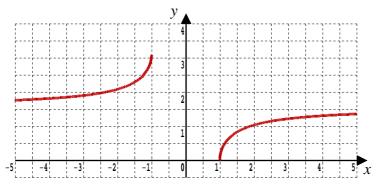
(i) It is obviously true that,  $\frac{1}{1+x^2} + \left(-\frac{1}{1+x^2}\right) = 0$ By integrating each term in this equation, deduce a simple relationship between  $\arctan x$  and  $\operatorname{arccot} x$ 

[3 marks]

( ii ) Hence, use the Maclaurin series for arctan x to deduce the Maclaurin series for arccot x.

# **Question 7**

With the aid of the graph, prove that the derivative of arcsec x is  $\frac{1}{x\sqrt{x^2-1}}$ 



The graph of y = arcsec x

[ **5** marks ]

# **Question 8**

Give a reason why a Maclaurin series for *arcsec x* will not exist.

[ 2 marks ]