Lesson 10

10.1 Revision

*Any solution based entirely on graphical
or numerical methods is not acceptable*

Marks Available : 40
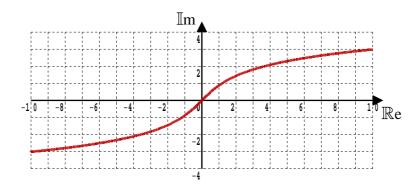
Question 1

(i)   Simplify $\left(\sqrt{x^2 + 1} - x \right)\left(\sqrt{x^2 + 1} + x \right)$

[1 mark]

(ii)   The graph is of the function $f(x) = ln\left(x + \sqrt{x^2 + 1} \right)$

Given that the graph would seem to have half turn rotational symmetry
about the origin does this suggest that the function is even or odd ?

[1 mark]

(iii)   Prove that the function is either an even function or an odd function,
as dictated by your answer to part (ii)

[4 marks]

Question 2

Show that $\dfrac{\cos 5\theta + \text{i} \sin 5\theta}{\cos 2\theta + \text{i} \sin 2\theta}$ can be expressed in the form $\cos n\theta + \text{i} \sin n\theta$,

where n is an integer to be found

[4 marks]

Question 3

Prove that $\dfrac{e^{\text{i}\theta} - 1}{e^{\text{i}\theta} + 1} = \text{i} \tan\left(\dfrac{x}{2}\right)$

[5 marks]

Question 4

Further A-Level Examination Question from June 2018, FP2, Q2(a) (MEI)

(i)   Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6\tan^2\theta + \tan^4\theta}{4\tan\theta\left(1 - \tan^2\theta\right)}$$

[5 marks]

(ii)   Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometric form

[4 marks]

Question 5

Further A-Level Examination Question from June 2009, FP2, Q3(b) (MEI)

The infinite series C and S are defined as follows,

$$C = \cos\theta + \frac{1}{3}\cos 3\theta + \frac{1}{9}\cos 5\theta + \ldots$$

$$S = \sin\theta + \frac{1}{3}\sin 3\theta + \frac{1}{9}\sin 5\theta + \ldots$$

By considering $C + iS$, show that $C = \dfrac{3\cos\theta}{5 - 3\cos 2\theta}$

and find a similar expression for S

Question 6

Given that $z = e^{\frac{\pi}{n}i}$ where n is a positive integer, prove that

$$1 + z + z^2 + \ldots + z^n = i \cot\left(\frac{\pi}{2n}\right)$$

[5 marks]