

Lesson 10

Further A-Level Pure Mathematics, Core 2 Complex Numbers II

10.1 Revision

*Any solution based entirely on graphical
or numerical methods is not acceptable*

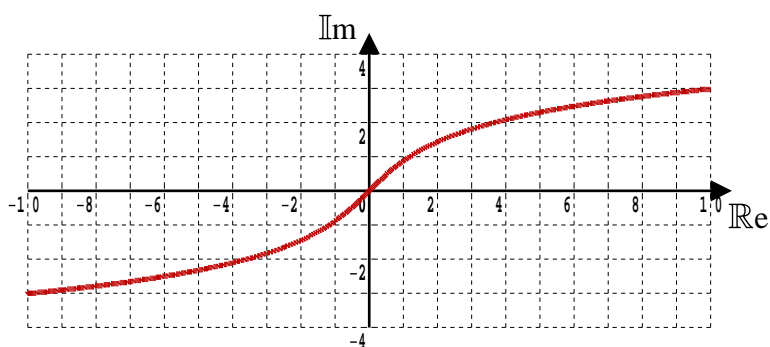
Marks Available : 40

Question 1

(i) Simplify $(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)$

[1 mark]

(ii) The graph is of the function $f(x) = \ln(x + \sqrt{x^2 + 1})$



Given that the graph would seem to have half turn rotational symmetry about the origin does this suggest that the function is even or odd ?

[1 mark]

(iii) Prove that the function is either an even function or an odd function, as dictated by your answer to part (ii)

[4 marks]

Question 2

Show that $\frac{\cos 5\theta + i \sin 5\theta}{\cos 2\theta + i \sin 2\theta}$ can be expressed in the form $\cos n\theta + i \sin n\theta$,
where n is an integer to be found

[4 marks]

Question 3

Prove that $\frac{e^{i\theta} - 1}{e^{i\theta} + 1} = i \tan\left(\frac{\theta}{2}\right)$

[5 marks]

Question 4

Further A-Level Examination Question from June 2018, FP2, Q2(a) (MEI)

- (i) Use de Moivre's theorem to prove that

$$\cot 4\theta = \frac{1 - 6 \tan^2 \theta + \tan^4 \theta}{4 \tan \theta (1 - \tan^2 \theta)}$$

- (ii) Hence express the roots of the equation

$$x^4 + 4x^3 - 6x^2 - 4x + 1 = 0$$

in exact trigonometric form

[5 marks]

[4 marks]

Question 5

Further A-Level Examination Question from June 2009, FP2, Q3(b) (MEI)

The infinite series C and S are defined as follows,

$$C = \cos \theta + \frac{1}{3} \cos 3\theta + \frac{1}{9} \cos 5\theta + \dots$$

$$S = \sin \theta + \frac{1}{3} \sin 3\theta + \frac{1}{9} \sin 5\theta + \dots$$

By considering $C + iS$, show that $C = \frac{3 \cos \theta}{5 - 3 \cos 2\theta}$

and find a similar expression for S

Question 6

Given that $z = e^{\frac{\pi}{n}i}$ where n is a positive integer, prove that

$$1 + z + z^2 + \dots + z^n = i \cot\left(\frac{\pi}{2n}\right)$$

[5 marks]

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Teachers may obtain detailed worked solutions to the exercises by email from mhh@shrewsbury.org.uk