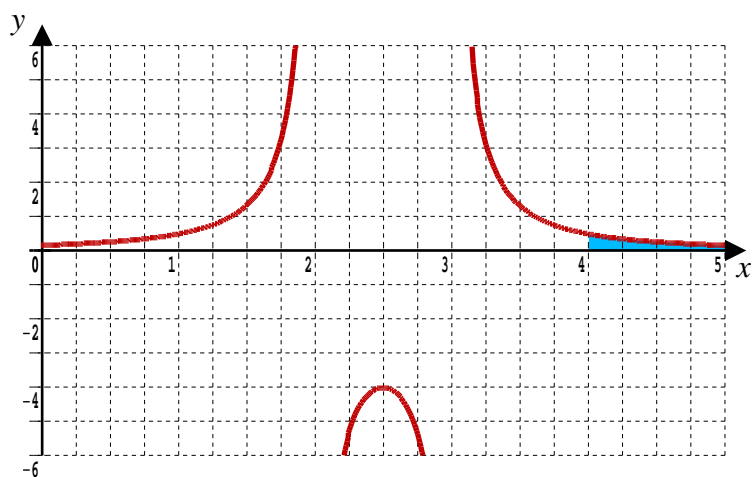


## Lesson 4

### Further A-Level Pure Mathematics, Core 2 Improper Integrals

#### 4.1 Partial Fractions

The graph is of the function  $f(x) = \frac{1}{x^2 - 5x + 6}$  and the task is to find the area from  $x = 4$  and extending rightward as  $x \rightarrow \infty$



Teaching video : <http://www.NumberWonder.co.uk/v9100/4.mp4>



[ 8 marks ]

## 4.2 Exercise

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 50

### Question 1

Use the method of partial fractions to show that  $\int_2^{\infty} \frac{1}{x^2 + x - 2} dx = \frac{2}{3} \ln 2$

[ 8 marks ]

**Question 2**

Use the method of partial fractions to show that  $\int_0^{\infty} \frac{19}{6x^2 + 35x + 36} dx = 3 \ln\left(\frac{3}{2}\right)$

[ 8 marks ]

### Question 3

- ( i )      Use the substitution  $x = 2 \tan \theta$  to show that,

$$\int \frac{1}{x^2 + 4} dx = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + c$$

[ 3 marks ]

- ( ii )      Show that  $\int_2^{\infty} \frac{32}{(x^2 + 4)(x + 2)} dx = \pi - 2 \ln 2$

[ 9 marks ]

**Question 4**

Show that  $\int_{\frac{1}{2}}^{\infty} \frac{1 - x^2}{4x^4 + x^2} dx = 2 - \frac{5\pi}{8}$

[ 10 marks ]

### Question 5

Show that  $\int_{2\sqrt{3}}^{\infty} \frac{3-x}{(x-2)^2(x^2+4)} dx = 2\pi + 3 + 3\sqrt{3} + 9 \ln\left(1 - \frac{\sqrt{3}}{2}\right)$

[ 12 marks ]

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In October 2020, Shrewsbury School was voted “**Independent School of the Year 2020**”

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Teachers may obtain detailed worked solutions to the exercises by email from [mhh@shrewsbury.org.uk](mailto:mhh@shrewsbury.org.uk)