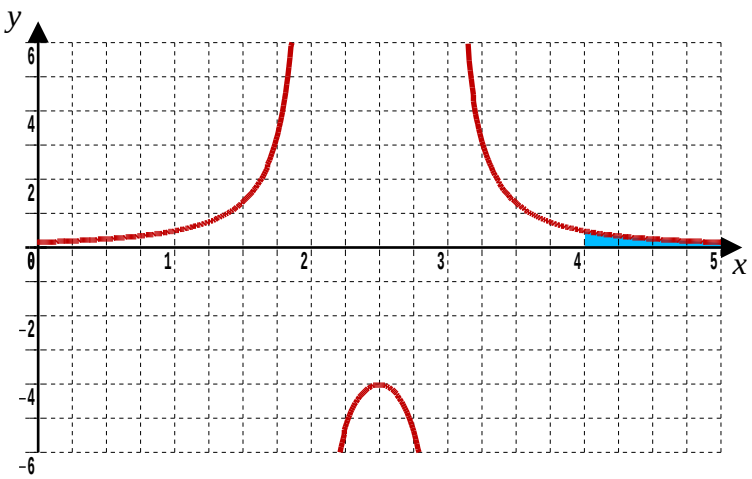
**Lesson 4**

**Further A-Level Pure Mathematics, Core 2**

**Improper Integrals**

## 4.1 Partial Fractions

The graph is of the function $f(x) = \dfrac{1}{x^2 - 5x + 6}$ and the task is to find the area from $x = 4$ and extending rightward as $x \to \infty$




Teaching video : http://www.NumberWonder.co.uk/v9100/4.mp4

**[ 8 marks ]**

**4.2 Exercise**

*Any solution based entirely on graphical or numerical methods is not acceptable*

Marks Available : 50

**Question 1**

Use the method of partial fractions to show that $\int_2^{\infty} \frac{1}{x^2 + x - 2}\, dx = \frac{2}{3} ln\, 2$

**[ 8 marks ]**

**Question 2**

Use the method of partial fractions to show that $\int_0^{\infty} \frac{19}{6x^2 + 35x + 36}\, dx = 3\, ln\left(\frac{3}{2}\right)$

**[ 8 marks ]**

**Question 3**

**( i )** Use the substitution $x = 2 \tan\theta$ to show that,

$$\int \frac{1}{x^2 + 4}\, dx = \frac{1}{2} \arctan\left(\frac{x}{2}\right) + c$$

**[ 3 marks ]**

**( ii )** Show that $\int_2^{\infty} \frac{32}{(x^2 + 4)(x + 2)}\, dx = \pi - 2 \ln 2$

**[ 9 marks ]**

**Question 4**

Show that $\int_{\frac{1}{2}}^{\infty} \frac{1 - x^2}{4x^4 + x^2}\, dx = 2 - \frac{5\pi}{8}$

**[ 10 marks ]**

**Question 5**

Show that $96\int_{2\sqrt{3}}^{\infty} \frac{3-x}{(x-2)^2(x^2+4)}\,dx = 2\pi + 3 + 3\sqrt{3} + 9\,ln\left(1 - \frac{\sqrt{3}}{2}\right)$

**[ 12 marks ]**