Lesson 6

Further A-Level Pure Mathematics, Core 2

Improper Integrals

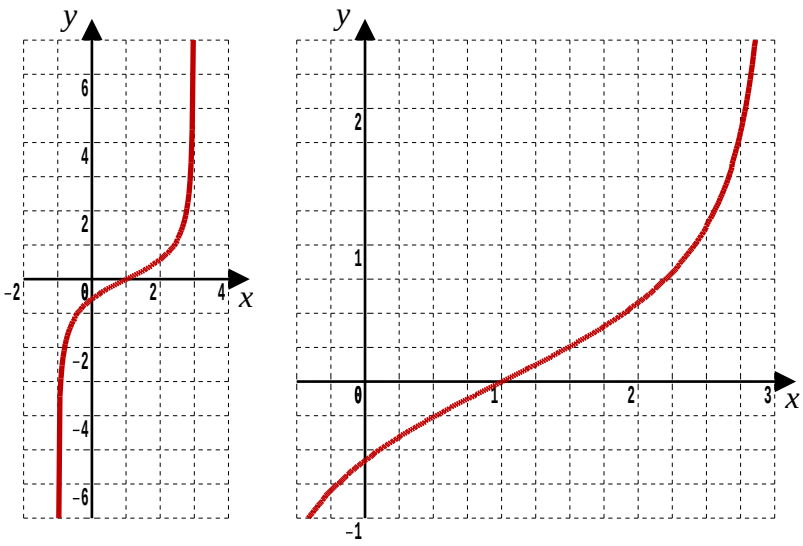
6.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40

Question 1

Two views of the graph of $f(x) = \dfrac{x - 1}{\sqrt{3 + 2x - x^2}}\,dx$ are presented below



Determine the value of the improper integral $\int_1^3 \dfrac{x - 1}{\sqrt{3 + 2x - x^2}}dx$

[6 marks]

Question 2

Further A-Level Examination Question from June 2016, FP2, Q1 (b), (OCR)

Find, in exact form, the value of the following integral,

$$\int_0^{\frac{3}{4}} \frac{1}{\sqrt{3 - 4x^2}}\, dx$$

Hint : Use the substitution $x = \frac{\sqrt{3}}{2} \sin\theta$

[6 marks]

Question 3

(i) Find $\int \frac{1}{x(x+5)}\,dx$

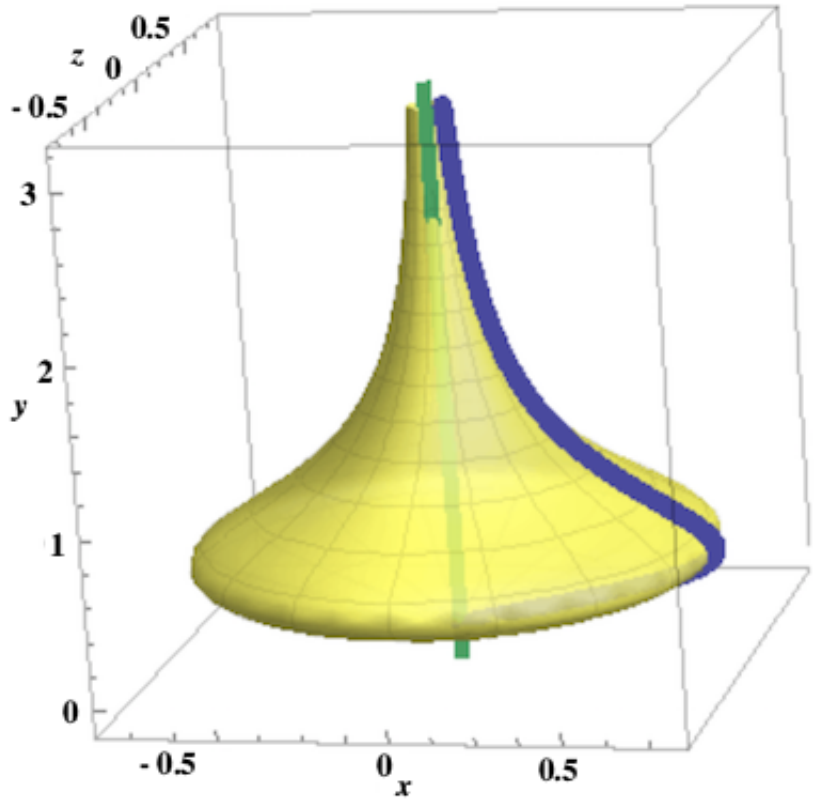
[4 marks]

(ii) Hence show that $\int_5^{\infty} \frac{1}{x^2+5x}\,dx$ converges and find its value

[3 marks]

Question 4

Find the volume swept out when the profile curve $x = \dfrac{y}{1 + 5y^3}$ is rotated $2\pi^c$ about the y-axis, $0 \leqslant y < \infty$. The lower end of the solid is depicted below.



[6 marks]

Question 5

Further A-Level Examination Question from January 2013, FP2, Q1(a) (OCR)

(i) Differentiate with respect to x the equation $a \tan y = x$ (where a is a constant) and hence show that the derivative of $\arctan\left(\frac{x}{a}\right)$ is $\frac{a}{a^2 + x^2}$

[3 marks]

(ii) By first expressing $x^2 - 4x + 8$ in completed square form, evaluate the integral $\int_0^4 \frac{1}{x^2 - 4x + 8}\, dx$ giving your answer exactly.

[4 marks]

(iii) Use integration by parts to find $\int arctan\, x\, dx$

[4 marks]

(iv) What does your part (iii) answer allow you to deduce regarding the improper integral $\int_0^{\infty} arctan\, x\, dx$? Give a reason for your answer.

[2 marks]

(v) Sketch the graph of $y = arctan\, x$ and explain how this backs up your part (iv) answer.

[2 marks]