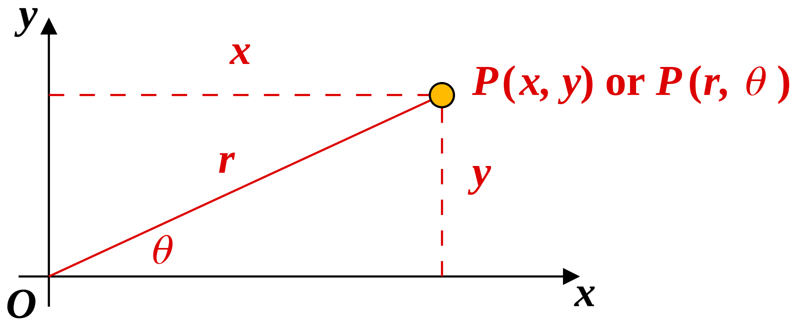
**Lesson 2**



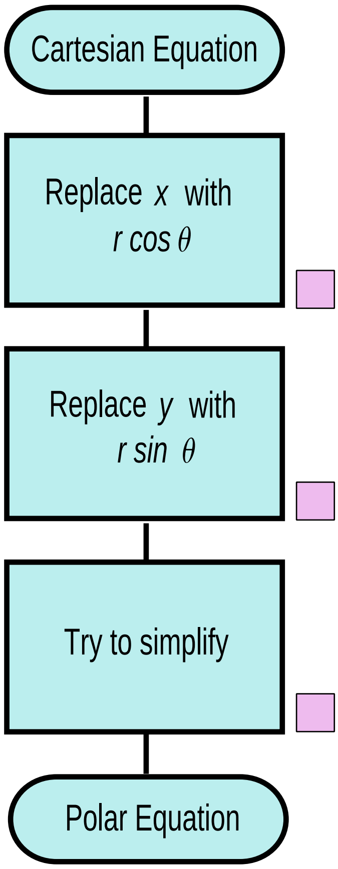
## 2.1 Curious and Interesting Curves

The key to moving between points and equations written in Cartesian form and the equivalent points and equations written in Polar form is the following diagram;




From the diagram, • $\cos\theta = \dfrac{x}{r}$ • $\sin\theta = \dfrac{y}{r}$ • $r^2 = x^2 + y^2$

This suggests the following strategy for manipulating the algebra of a Cartesian equation to recast it in Polar form.

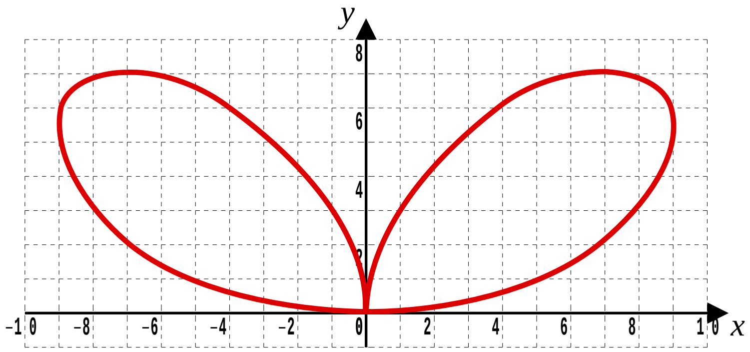



"Try to simplify" is vague; the ideal output is an equation of the form $r = f(\theta)$ or $r^2 = f(\theta)$ but that may not be possible.

## 2.2 Example

The famous curve below is called The Double Follium. It has Cartesian equation,

$$(x^2 + y^2)^2 = 28x^2y$$




Find the polar equation of The Double Follium.

**[ 4 marks ]**

**Solution**

A shortcut is to spot that the $x^2 + y^2$ piece of the equation can be replaced with $r^2$ alongside the more usual replacement of $x$ with $r\cos\theta$ and $y$ with $r\sin\theta$

$$(x^2 + y^2)^2 = 28x^2y$$

$$(r^2)^2 = 28\,(r\cos\theta)^2(r\sin\theta)$$

$$r^4 = 28r^3\cos^2\theta\sin\theta$$

$$r = 28\cos^2\theta\sin\theta$$

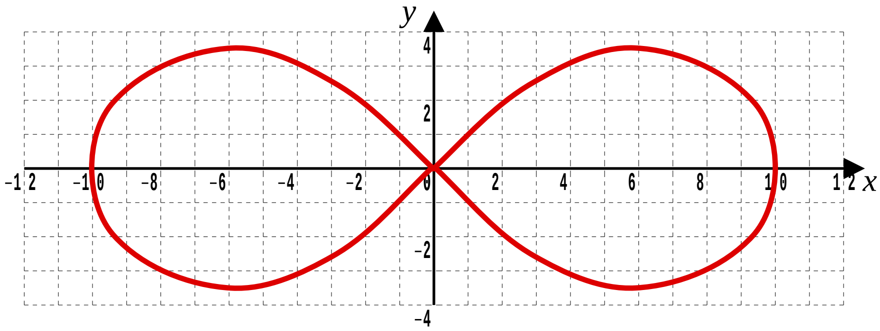
## 2.3 Exercise

*Any solution based entirely on graphical*
*or numerical methods is not acceptable*
Marks Available : 40

**Question 1**

The curve below is called The Lemniscate of Bernoulli and has Cartesian equation

$$\left(x^2 + y^2\right)^2 = 100\left(x^2 - y^2\right)$$



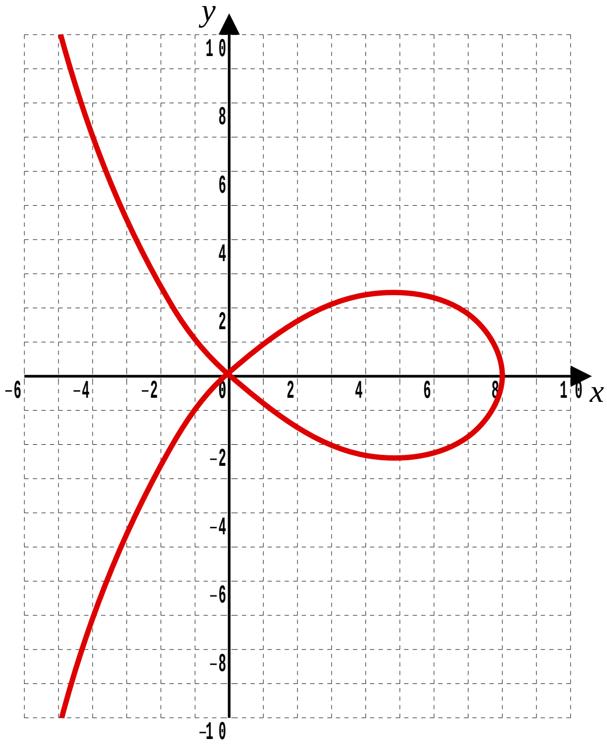

Show that the polar equation of The Lemniscate of Bernoulli can be written in the form $r^2 = 100\,cos\left(2\theta\right)$

**[ 3 marks ]**

**Question 2**

The curve below is a Right Strophoid and has Cartesian equation

$$y^2(8+x) = x^2(8-x)$$




**( i )** Show that the Right Strophoid has polar equation $r = \dfrac{8\cos(2\theta)}{\cos\theta}$

**[ 6 marks ]**

**( ii )** Show that the point with Cartesian coordinates $\left(-4, -4\sqrt{3}\right)$ is on the curve.

**[ 2 marks ]**

**( iii )** By means of implicit differentiation of the Cartesian equation show that

$$\frac{dy}{dx} = \frac{16x - 3x^2 - y^2}{16y + 2xy}$$

**[ 4 marks ]**

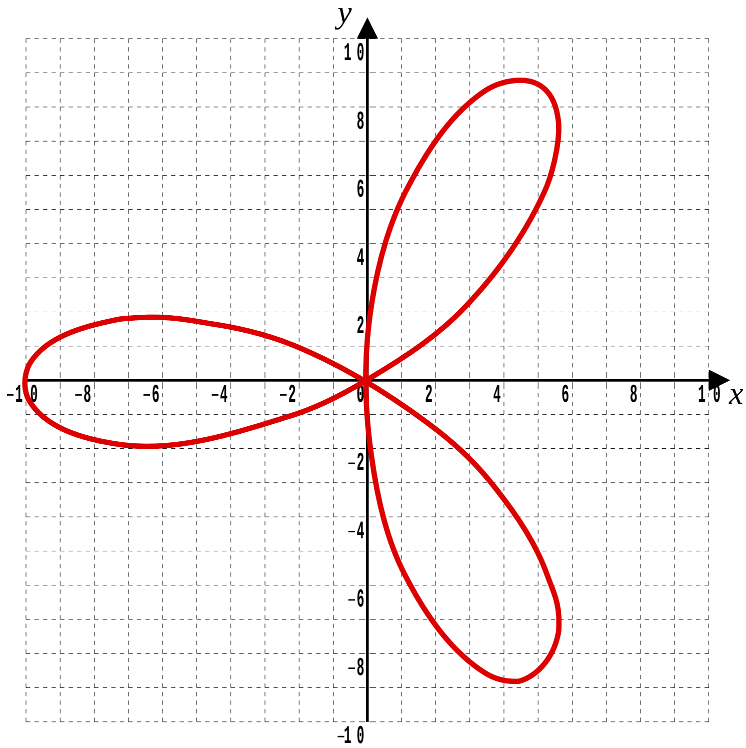
**( iv )** Find the equation of the tangent to the curve at the point $\left(-4, -4\sqrt{3}\right)$ in the form $y = mx + c$ where $m$ and $c$ are irractional constants to be found.

**[ 3 marks ]**

**Question 3**

The curve below is a Trifolium (three-loop) and has Cartesian equation

$$\left(x^2 + y^2\right)\left(y^2 + x\left(x + 10\right)\right) = 40\,x\,y^2$$




**( i )** Show that the Trifolium has polar equation $r = 10\cos\theta\left(4\sin^2\theta - 1\right)$

**[ 5 marks ]**

**( ii )** Find the value of $r$ when $\theta = 60°$

**[ 1 mark ]**

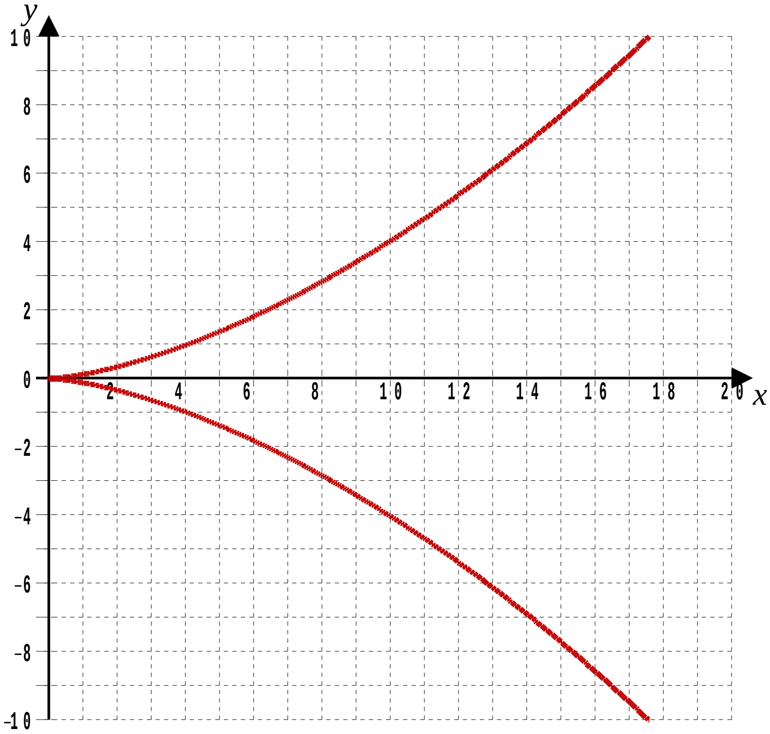
**( iii )** Find the value of $r$ when $\theta = 30°$ and deduce the equation of the tangent to the curve for this value of $\theta$

**[ 3 marks ]**

**Question 4**

The curve below is The Cissoid of Diocles and has Cartesian equation

$$y^2 = \frac{x^3}{72 - x}$$




**( i )** Show that The Cissoid of Diocles has polar equation $r = 72 \tan \theta \sin \theta$

**[ 4 marks ]**

**( ii )** Show that the point with Cartesian coordinates $\left(8,\ 2\sqrt{2}\right)$ is on the curve

**[ 2 marks ]**

**( iii )** By means of implicit differentiation of the Cartesian equation show that

$$\frac{dy}{dx} = \frac{3x^2 + y^2}{144y - 2xy}$$

**[ 4 marks ]**

**( iv )** Find the equation of the tangent to the curve at the point $\left(8,\ 2\sqrt{2}\right)$ in the form $y = a\sqrt{2}\,x + b\sqrt{2}$ where $a$ and $b$ are rational numbers.

**[ 3 marks ]**