



How well is your maths today ?

*Any solution based entirely on graphical  
or numerical methods is not acceptable*

Marks Available : 30

**Question 1**

Solve the equation  $\sin(x + 30^\circ) = 2 \cos x$ , for  $0 \leq x \leq 360^\circ$

Give your answers as exact values.

[ 5 marks ]

**Question 2**

$f(x) = 4x^3 + ax^2 + 20x + 4$  where  $a$  is an integer constant.

Given that  $(4x + 1)$  is a factor of  $f(x)$  find the value of  $a$

[ 5 marks ]

**Question 3**

( i ) Given that,  $x \in \mathbb{R}$  and  $g(x) = x^3 - 12x^2 + 50x - 7$ , find  $g'(x)$

[ 2 marks ]

( ii ) By completing the square, show that  $g(x)$  is an increasing function

[ 5 marks ]

**Question 4**



Squirrels were introduced into a forest. The number of squirrels,  $S$ , in the forest  $T$  years after they were introduced is modelled by the equation

$$S = \frac{390}{1 + e^{4-1.4T}} \quad T \in \mathbb{R}, T \geq 0$$

( i )      How many squirrels were initially introduced into the forest ?

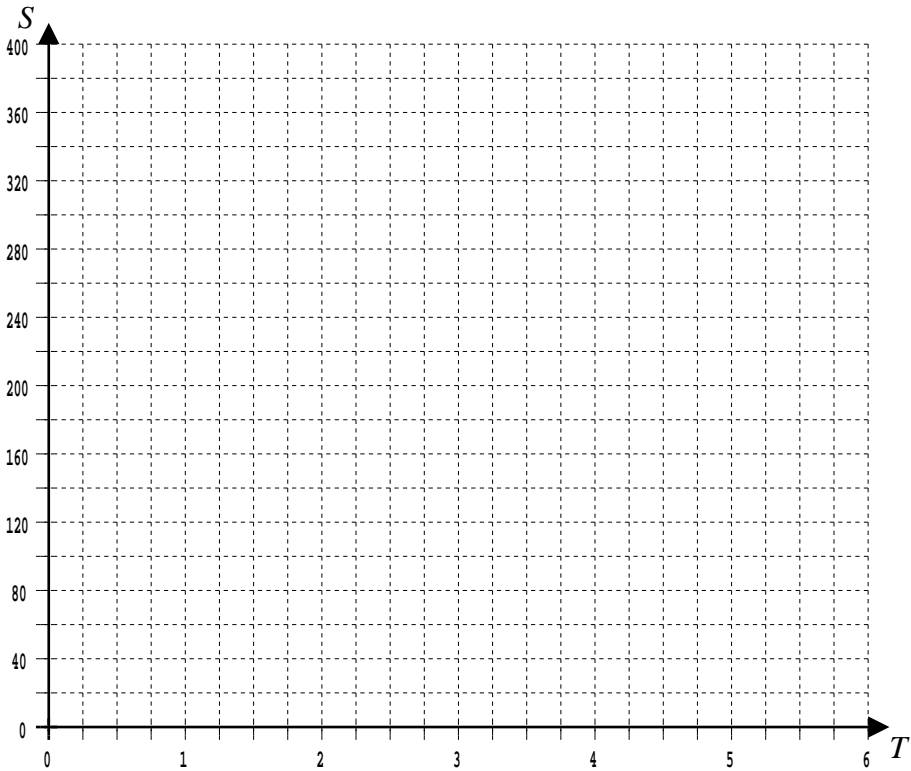
[ 1 mark ]

( ii )      Complete the following table to show how the model suggests the squirrel population will grow over the subsequent 6 years.

$T$ (years)	0	1	2	3	4	5	6
$S$ (N° of Squirrels)							

[ 2 marks ]

( iii )      On the graph below plot the squirrel's population growth curve



[ 2 marks ]

- ( iv ) George claims that, looking further into the future, the model predicts that the squirrel population will grow exponentially, because the equation contains an exponential.

State if you agree with George or not.

[ 1 mark ]

Either way, give a mathematical reason for the future overall trend of the squirrel population, as predicted by the model.

[ 2 marks ]

### Question 5

Solve the following, giving your answer to 3 significant figures,

$$2^{2x} + 3(2^{x-1}) - 1 = 0$$

[ 5 marks ]

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Teachers may obtain detailed worked solutions to the exercises by email from MHHShrewsbury@Gmail.com