Year 2 **Pure Mathematics Examination Revision : Health Check N° 11**

Who is the coolest doctor in the hospital ?
The hip consultant !

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 47

Question 1

Given $y = x(2x + 1)^4$, show that $\frac{dy}{dx} = (2x + 1)^n(Ax + B)$ where n, A and B are constants to be found.

[4 marks]

Question 2

(i) The fourth term of a geometric series is 16 and the seventh term of the series is 250. Find both the common ratio and the first term.

[4 marks]

(ii) The third, fourth and fifth terms of an arithmetic sequence are $3k$, $4k + 3$ and $6k - 9$ respectively, where k is a constant.
Show that the sum of the first n terms of the sequence is a square number.

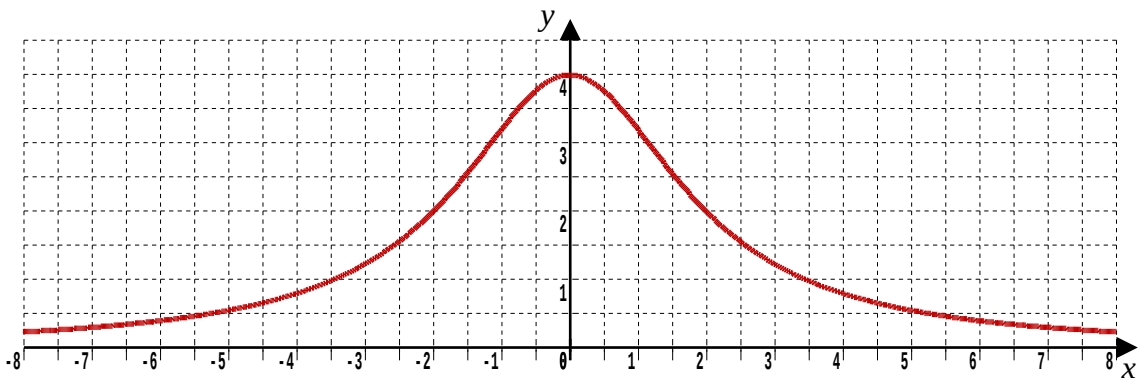
[5 marks]

(iii) The first term of a geometric progression is 4 and the sum to infinity is 12. Find the common ratio.

[3 marks]

Question 3

The graph is of the function, $f(x) = \dfrac{16}{4 + x^2} \quad x \in \mathbb{R}$



(a) Find the exact coordinates of the two points of inflection

[6 marks]

The function m and the function n are defined by

$$m(x) = \frac{1}{4} f(x) \qquad x \in \mathbb{R}$$

$$n(x) = \frac{1}{3} f(x) + 4 \qquad x \in \mathbb{R},\ x \geqslant \frac{2\sqrt{3}}{3}$$

(b) Find **(i)** the range of m

[1 mark]

(ii) the range of n

[2 marks]

Question 4

(a) Express $5\cos\theta - 8\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \pi$. Write R in surd form and give α to 4 decimal places.

[4 marks]

The temperature of a kiln, T °C, used to fire pottery, can be modelled by,

$$T = 1100 + 5\cos\left(\frac{x}{3}\right) - 8\sin\left(\frac{x}{3}\right), \qquad 0 \leqslant x \leqslant 72$$

where x is the time in hours since the pottery was placed in the kiln.

(b) Calculate the maximum value of T predicted by this model and the value of x, to 2 decimal places, when this maximum first occurs.

[4 marks]

(c) Calculate the times during the first 24 hours when the temperature is predicted, by this model, to be exactly 1097 °C

[4 marks]

Question 5

The value, £V, of a vintage car t years after it was first valued on 1st January 2001 is modelled by the equation,

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005
and £50 000 on 1st January 2012

(a) **(i)** find p to 4 decimal places

(ii) show that A is approximately 24 800

[4 marks]

(b) With reference to the model, interpret

(i) the value of the constant A

(ii) the value of the constant p

[2 marks]

Using the model.

(c) find the year during which the value of the car first exceeds £100 000

[4 marks]