Year 2 **Pure Mathematics Examination Revision : Health Check N° 9**

How did the doctor cure the invisible man ?
The doctor took him to the ICU !

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 30

Question 1

Find the exact solutions to the equation

$$e^x + 12\,e^{-x} = 7$$

[4 marks]

Question 2

Given that $|x| < \frac{1}{6}$ find the binomial expansion of $\sqrt{1 - 6x}$ in ascending powers of x up to and including the term in x^3, simplifying each term.

[4 marks]

Question 3

Prove that $\csc\theta \sec^2\theta = \csc\theta + \tan\theta \sec\theta$

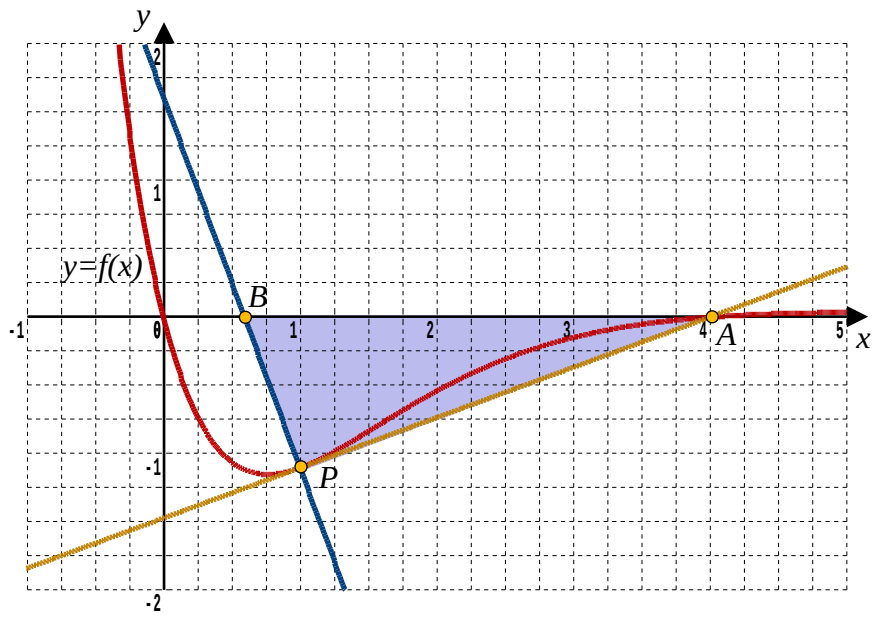
[4 marks]

Question 4

The graph is of the curve $y = f(x)$, where $f(x) = \left(x^2 - 4x\right) e^{-x}$

The point P has coordinates $\left(1, -\frac{3}{e}\right)$

The tangent to the curve at P intersects the x-axis at the point A.



(i) Find the (exact) equation of the tangent.

[5 marks]

The normal to the curve at P intersects the x-axis at the point B.

(ii) Show that the area of the triangle ABP is $\frac{9\left(e^2 + 1\right)}{2e^3}$

[8 marks]

Question 5

Sheldon's grandfather gives him £10 on his first birthday.
He says that he will give him £20 on his second birthday, £40 on his third, and so on, doubling his gift each year until Sheldon is 18.
If he goes through with his plan,

(i) How much will he have to give to Sheldon on his eighteenth birthday ?

[2 marks]

(ii) How much in total will he have given Sheldon by (and including) his eighteenth birthday ?

[3 marks]