Do NOT open this paper until instructed to do so. **While you are waiting to start write your name in the box directly below.**
Name _____ Set _____

SHREWSBURY SCHOOL

ADDITIONAL MATHEMATICS
PAPER 1

Constructed from recent official OCR examination resources (6993)

Thursday 6th May 2021

8.30 am – 9.50 am (80 minutes)
Extra Time finish : 10.10 am

5m1 & 5m2

- ◇ There are 60 marks available in this paper.
- ◇ You should attempt as many questions as you can.
- ◇ You must show full working, where appropriate, in order to gain full marks.
- ◇ You are expected to use a calculator in this paper.

Formulae
FSMQ Additional Maths (6993)

Binomial series

$$(a + b)^n = a^n + {}^nC_1\, a^{n-1} b + {}^nC_2\, a^{n-2} b^2 + ... + {}^nC_r\, a^{n-r} b^r + ... + b^n$$

for positive integers, n, where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$, $r \leqslant n$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$

Numerical methods

Trapezium rule: $\int_a^b y\, dx = \frac{1}{2} h\left(\left(y_0 + y_n\right) + 2\left(y_1 + y_2 + ... + y_{n-1}\right)\right)$

Kinematics

Variable acceleration formulae

$$v = \frac{ds}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

$$s = \int v\, dt \text{ and } v = \int a\, dt$$

Constant acceleration formulae

$$v = u + at$$

$$s = ut + \frac{1}{2} a t^2$$

$$s = \frac{1}{2} (u + v)\, t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2} a t^2$$

Question 1

You are given that $y = x^3 + 2x - 7$

(a) Find $\frac{dy}{dx}$

[2 marks]

(b) Use your result to part (a) to show that the graph of $y = x^3 + 2x - 7$ has no turning points

[2 marks]

Question 2

Show that $\frac{2}{3 + \sqrt{2}}$ can be written in the form $\frac{a - \sqrt{b}}{c}$

where *a*, *b* and *c* are integers to be found

[3 marks]

Question 3

(a) Simplify $\frac{x}{x+2} - \frac{6}{x-1}$

[3 marks]

In this question you must show detailed reasoning

(b) Solve $\frac{x}{x+2} - \frac{6}{x-1} = 4$ giving your answer in exact form.

[4 marks]

Question 4
In this question you must show detailed reasoning

(a) Express $2x^2 + 8x - 12$ in the form $a(x + p)^2 + q$

[4 marks]

(b) Hence find the minimum value of $2x^2 + 8x - 12$

[1 mark]

Question 5

Solve $sin(x + 30°) = 0.2$ for $0° \leqslant x \leqslant 360°$

[4 marks]

Question 6

In this question you must show detailed reasoning.

(a) Show that $(x - 3)$ is a factor of $x^3 - 5x^2 + x + 15$

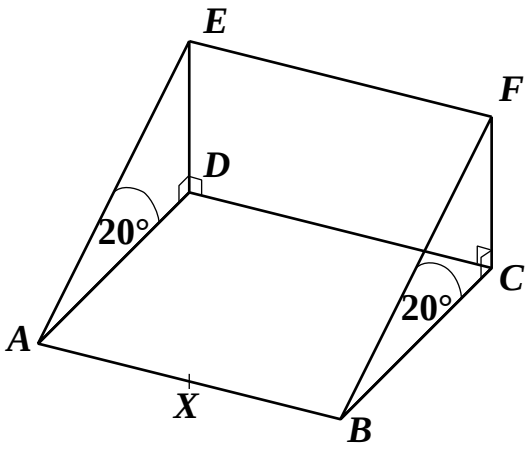
[1 mark]

(b) Hence solve the equation $x^3 - 5x^2 + x + 15 = 0$

Give exact answers.

[4 marks]

Question 7



The diagram shows a triangular prism.
The rectangle *ABCD* is horizontal and *ABFE* is a square inclined at 20° to the horizontal such that *E* is vertically above *D* and *F* is vertically above *C*.
The area of the square *ABFE* is 1600 m^2.
X is a point on *AB* such that *AX* = 12 m.

Calculate

(a) the area of *ABCD*

[3 marks]

(b) the angle between the lines *XE* and *XF*

[5 marks]

Question 8

An object is falling through a liquid. The distance fallen is modelled by the formula $s = 48t - t^3$ until it comes to rest, where s is the distance fallen in centimetres and t is the time in seconds measured from the point when the object entered the liquid.

(a) Find

(i) the acceleration when $t = 1$

[5 marks]

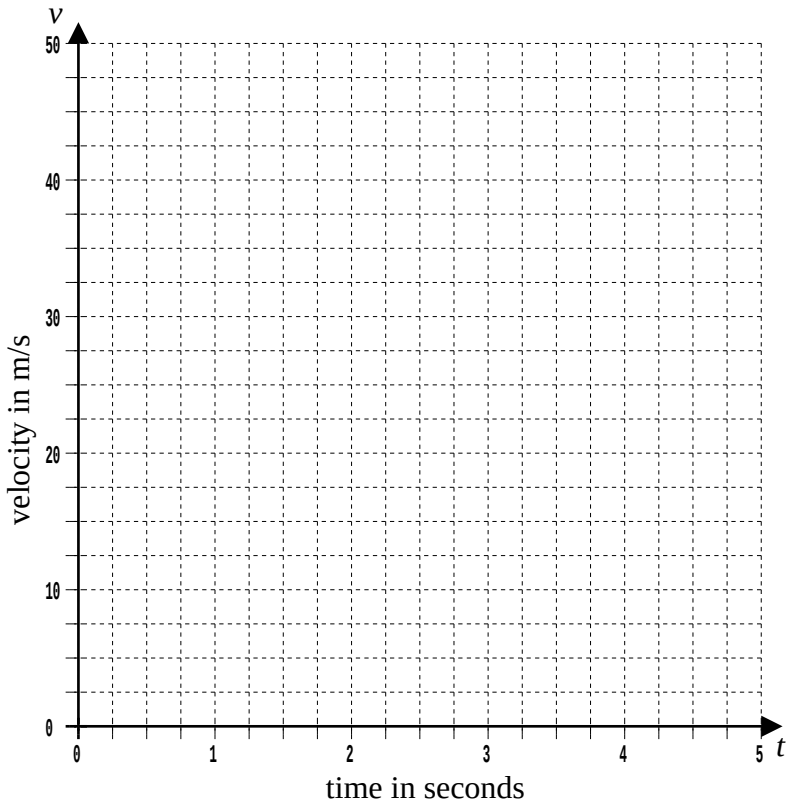
(ii) the time when the object comes to rest

[2 marks]

(iii) the distance fallen when the object comes to rest

[2 marks]

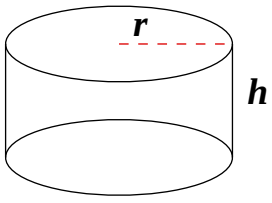
(b) Sketch the velocity/time graph for the period of time until the object comes to rest.



[1 mark]

Question 9

A container with volume 16π cm^3 is to be a closed cylinder.
The manufacturer wishes to minimise the surface area of the cylinder.



(i) Show that $h = \frac{16}{r^2}$ where h is the height (in cm) of the cylinder and r is the radius (in cm) of the circular base.

[1 mark]

(ii) Hence show that the surface area is given by $S = 2\pi r^2 + \frac{32\pi}{r}$

[3 marks]

(iii) Find the value of r when $\frac{dS}{dr} = 0$

[5 marks]

(iv) Find the value of $\frac{d^2S}{dr^2}$ when $\frac{dS}{dr} = 0$

[4 marks]

(v) Hence, find the minimum surface area of the cylinder

[1 mark]

[End Of Paper]