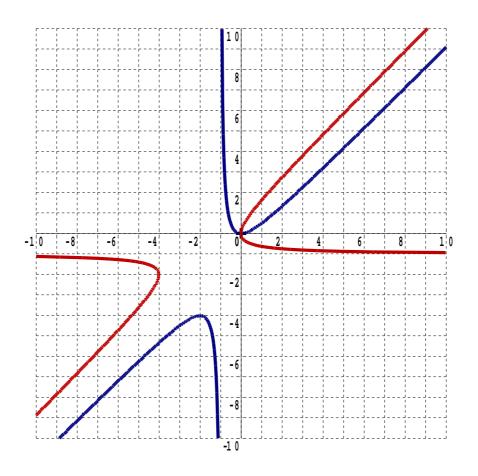
A-Level Pure Mathematics

Year 1 Graph Work

featuring

Graph Transformations

 $G \bullet R \bullet A \bullet P \bullet H \quad W \bullet O \bullet R \bullet K$



The graphs of
$$y = \frac{x^2}{x+1}$$
 (in blue) and $x = \frac{y^2}{y+1}$ (in red)

Each is a reflection of the other in the line y = x

1.1 Graphing Polynomials

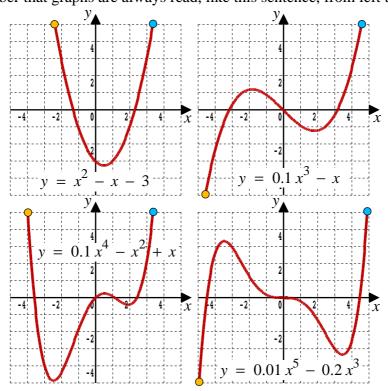
Faced with the task of quickly sketching a polynomial how best to proceed? It turns out that the degree of the polynomial is of crucial importance.

The Degree of a Polynomial

The degree of a polynomial the highest power of x.

1.2 Graphed Polynomials with Positive Highest Order Term

Here are a few examples of polynomials and their corresponding graphs. For each, note the parity (odd or even) of the degree of the polynomial and, for large values of x, which quadrant the graph enters and exits. (Remember that graphs are always read, like this sentence, from left to right)



Here is a summary of the key observations to be made,

Equation	Degree	Degree's Parity	Entry	Exit
$y = x^2 - x - 3$	2	even	2 nd	1 st
$y = 0.1 x^3 - x$	3	odd	3 rd	1 st
$y = 0.1 x^4 - x^2 + x$	4	even	2 nd	1 st
$y = 0.01 x^5 - 0.2 x^3$	5	odd	3 rd	1 st

The observations made suggest the first half of the following rule.

Large Values of *x* **Rule for Polynomials**

If the highest order term is positive then the exit is in the 1st quadrant. Additionally,

- if the degree is even the entry is in the 2^{nd} quadrant.
- if the degree is odd the entry is in the 3rd quadrant.

If the highest order term is negative then the exit is in the 4th quadrant. Additionally,

- if the degree is even the entry is in the 3rd quadrant.
- if the degree is odd the entry is in the 2nd quadrant

1.3 Example

$$f(x) = x^4 + 4x^3 - 17x^2 - 60x$$

Given that (x - 4) is a factor, factorise f(x) completely and hence sketch the graph of f(x) clearly marking all axis crossing points.

1.4 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available: 74

Question 1

$$f(x) = x^4 + 2x^3 - 11x^2 - 12x$$

Given that (x - 3) is a factor, factorise f(x) completely and hence sketch the graph of f(x) clearly marking all axis crossing points.

$$f(x) = 6x^3 + 7x^2 - 23x - 30$$

(i) Calculate the value of f(1), the value of f(2), and the value of f(3)

[3 marks]

The Factor Theorem

If, for a given polynomial function p(x), p(a) = 0 (for some constant, a) then (x - a) is a factor of p(x)

(ii) Hence, using the factor theorem, factorise f(x) completely and so sketch the graph of f(x) clearly marking all axis crossing points.

$$p(x) = a^2 - x^2$$
 where a is a positive constant

Sketch the graph of p(x) clearly making all axis crossing points in terms of a.

[3 marks]

Question 4

$$q(x) = x^2 + a^2$$
 where a is a positive constant

Sketch the graph of q(x) clearly making all axis crossing points in terms of a.

[3 marks]

Question 5

$$r(x) = x^4 - a^4$$
 where a is a positive constant

Sketch the graph of r(x) clearly making all axis crossing points in terms of a.

$$f(x) = 7x^3 - 36x - x^4$$

Given that (x + 2) is a factor, factorise f(x) completely and hence sketch the graph of f(x) clearly marking all axis crossing points.

A-Level Examination Question from June 2019, Paper 1, Q1 (Edexcel)

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that (x + 3) is a factor of f(x) find the value of the constant a

[3 marks]

Question 8

Continue the previous question by factorising f(x) completely, using the value of a found, and so sketch the graph of f(x) marking on its two axis crossing points.

Given that $f(x) = (x - 10)(x^2 - 2x) + 12x$

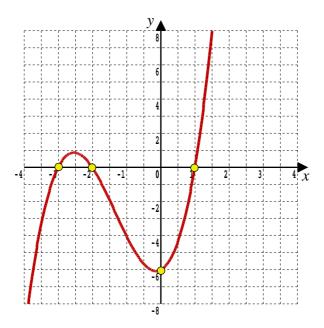
(i) Express f(x) in the form $x(ax^2 + bx + c)$ where a, b and c are real constants

[3 marks]

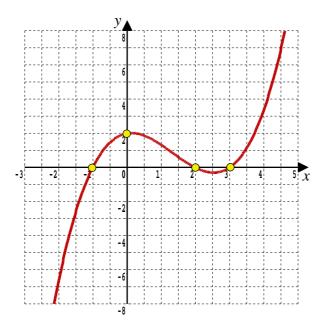
(ii) Hence factorise f(x) completely.

[2 marks]

(iii) Sketch the graph of y = f(x) showing clearly the points where the graph intersects the axes.



The graph is of $y = x^3 + bx^2 + cx + d$, where b, c and d are real constants. Find the values of b, c and d.



The graph is of $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are real constants. Find the values of a, b, c and d.

Question	12
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$$f(x) = x^4 + 6x^3 + x^2 - 24x + 16$$

(i) Given that the roots of f(x) have integer values, list the ten possible roots suggested by the term independent of x being 16.

[2 marks]

(ii) Use the factor theorem to find two roots of f(x)

[2 marks]

(iii) Hence, factorise f(x) completely and so sketch the graph of f(x) clearly marking all axis crossing points.

[8 marks]