Lesson 7

A-Level Pure Mathematics : Year 2

Integration III

7.1 Sweating the Exam

Example

This question is about finding the area in the quadrant between the positive x-axis and the positive y-axis and the curve, C, with parametric equations,

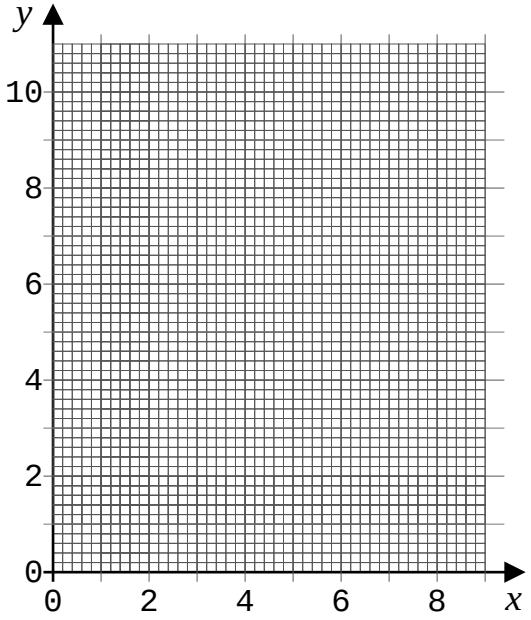
$$x = 4 - t^2 \text{ and } \quad y = t\,(t + 1) \quad \text{for} \quad t \geqslant 0$$

(i) Complete the following table,

t	0	0.5	1	1.5	2
x					
y					

[2 marks]

(ii) Use the table to sketch the curve, C, and shade the area to be found.



[2 marks]

(iii) Use parametric integration to find the area.
The following teaching video demonstrates a method of solution, and may be used to help write out your solution, if required.
http://www.NumberWonder.co.uk/Video/v9045(11a).mp4

Notice that, as the bend is anticlockwise as t increases, strictly speaking the integration gives the area sought but with a negative sign. The video has a convincing dodge to argue about which is the lower and which is the upper limit. The following rule is worth keeping in mind;

The Limit Swap Rule

$$\int_A^B -f(x)\,dx = \int_B^A f(x)\,dx$$

The video begins having already done the following step,

$$Area = \int y\,dx = \int y\,\frac{dx}{dt}\,dt$$

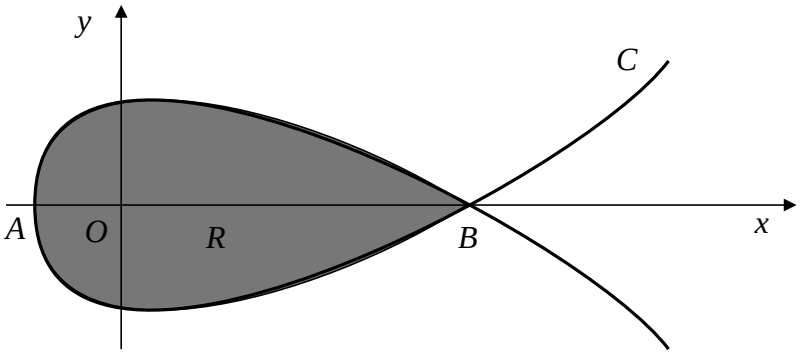
[6 marks]

7.2 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 34

Question 1

A-Level Examination Question from January 2010, Paper C4, Q7 (Edexcel)



The diagram shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4 \qquad y = t\left(9 - t^2\right)$$

The curve C cuts the x-axis at the points A and B

(a) Find the x-coordinate at the point A and the x-coordinate at the point B

[3 marks]

The region R, shown shaded, is enclosed by the loop of the curve

(b) Use integration to find the area of R

[6 marks]

Help for Q1 : http://www.NumberWonder.co.uk/Video/v9045(11b).mp4

Question 2

A-Level Examination Question from June 2018, Q14 (Edexcel)

A curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = 4 + 2\cos 2t \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy

$$y = 6 - (x - 3)^2$$

[2 marks]

(b) **(i)** Sketch the curve C

(ii) Explain briefly why C does not include all points of

$$y = 6 - (x - 3)^2 \qquad x \in \mathbb{R}$$

[3 marks]

The line with equation $x + y = k$ where k is a constant, intersects C at two distinct points

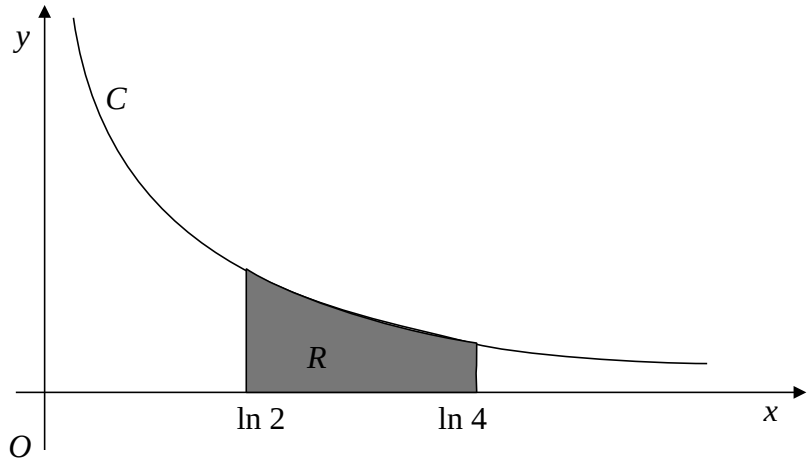
(c) State the range of values of k writing your answer in set notation

[5 marks]

Help for Q2 : http://www.NumberWonder.co.uk/Video/v9045(11c).mp4

Question 3

A-Level Examination Question from January 2008, Paper C4 (Edexcel)



The curve C has parametric equations

$$x = ln\,(t + 2) \qquad y = \frac{1}{(t + 1)}$$

The finite region R between the curve C and the x-axis, bounded by the lines with equations $x = ln\,2$ and $x = ln\,4$, is shown shaded.

(a) Show that the area of R is given by the integral,

$$\int_0^2 \frac{1}{(t + 1)\,(t + 2)}\,dt$$

[4 marks]

(b) Hence find an exact value for this area.

[6 marks]

(c) Find a Cartesian equation of the curve C in the form $y = f(x)$

[4 marks]

(d) State the domain of values for x for this curve.

[1 mark]

Help for Q3 : http://www.NumberWonder.co.uk/Video/v9045(11d).mp4