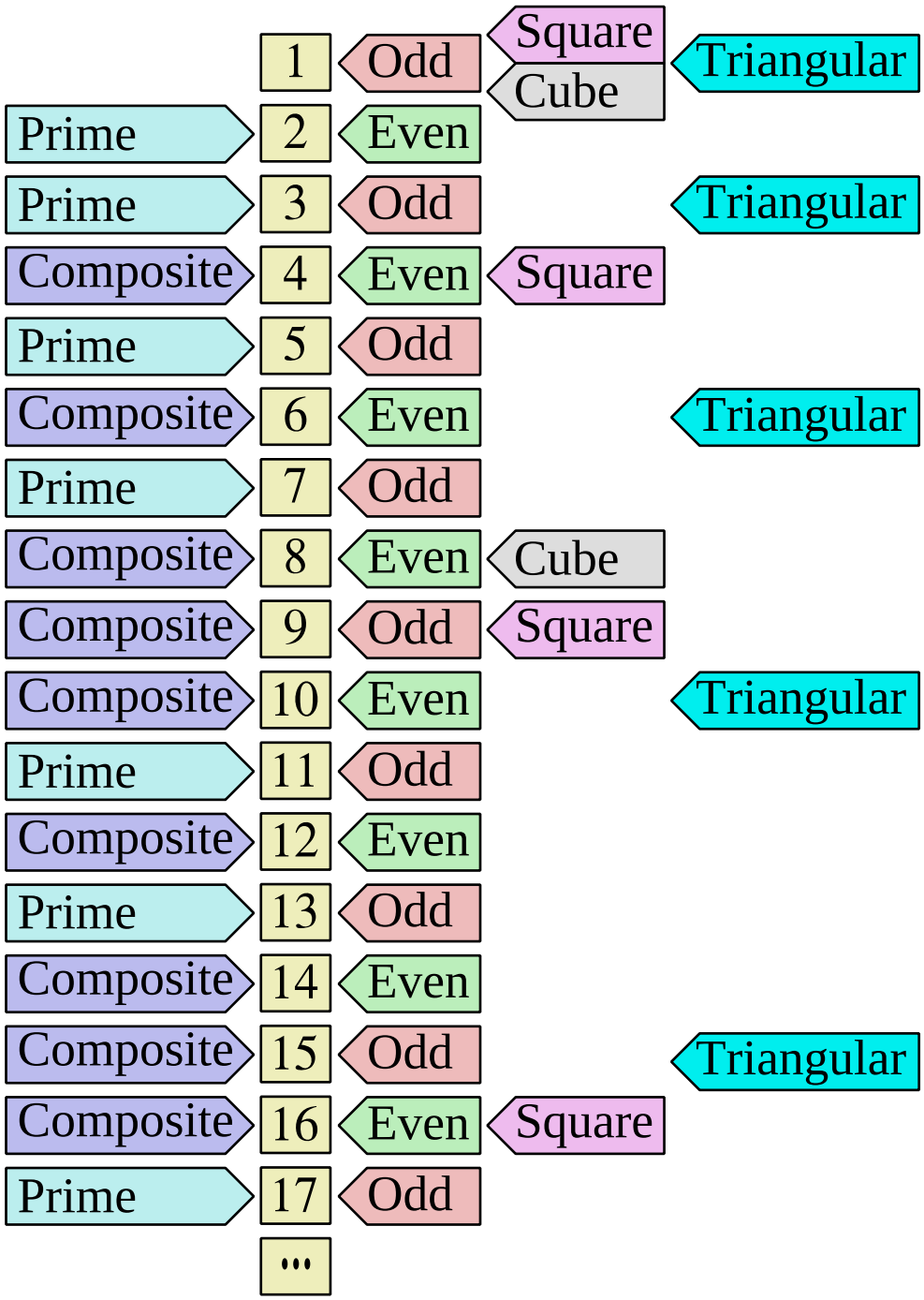
A-Level

~ Year 1 ~

Pure Mathematics

PROOF

~ The Art of Absolute Certainty ~



P R O O F
~ The Art of Absolute Certainty ~

Lesson 1

A-Level Pure Mathematics, Year 1
Proof I : The Art of Absolute Certainty

1.1 Consecutive Numbers

Our first foray into proof is going to be mostly working with the natural numbers, $\mathbb{N}$, which are often referred to as "The Counting Numbers",

$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, ...\}$

Notice that zero is not included.

1.2 A Consecutive Sum Example

Three consecutive natural numbers have a sum of 84.
What are the three consecutive natural numbers ?

Method 1 : Form and Solve an Equation

[2 marks]

Method 2 : Make a Simplifying Assumption

[2 marks]

1.3 A Consecutive Product Example

Three consecutive natural numbers have a product of 85140.
What are the three consecutive natural numbers ?

Method 1 : Form and Solve an Equation

[2 marks]

Method 2 : Make a Simplifying Assumption

[2 marks]

1.4 Consecutive Odd Numbers

Complete the following table for the odd number sequence

p	1	2	3	4	...	n	$n + 1$	$n + 2$	...
T_p	1	3			...				...

[4 marks]

1.5 A First Proof

(a) Calculate each of the following,

(i) 1×3

(ii) 3×5

(iii) 5×7

(iv) 7×9

(v) 9×11

[2 marks]

(b) Prove that the product of two consecutive odd numbers is always one less than a square number.

[4 marks]

1.6 Exercise

Any solution based entirely on graphical or numerical methods is not acceptable

Marks Available : 40 marks

Question 1

Three consecutive natural numbers have a sum of 204
What are the three consecutive natural numbers ?

[2 marks]

Question 2

Three consecutive natural numbers have a product of 778596
What are the three consecutive natural numbers ?

[2 marks]

Question 3

Four consecutive natural numbers have a sum of 78
What are the four consecutive natural numbers ?

[2 marks]

Question 4

Five consecutive natural numbers have a product of 95040
What are the five consecutive natural numbers ?

[3 marks]

Question 5

(a) Calculate each of the following,

(i) $3^2 - 1^2$

(ii) $5^2 - 3^2$

(iii) $7^2 - 5^2$

(iv) $9^2 - 7^2$

(v) $11^2 - 9^2$

[3 marks]

(b) Prove that the difference between two consecutive odd numbers that have been squared is always divisible by 8

[5 marks]

Question 6

Prove that the sum of four consecutive odd numbers is always divisible by 8

[4 marks]

Question 7

Complete the following table for the even number sequence,

p	1	2	3	4	...	$n-1$	n	$n+1$	...
T_p	2	4			...				...

[5 marks]

Question 8

Three consecutive even numbers have a sum of 72.
What are the three consecutive even numbers ?

[4 marks]

Question 9

Prove that the sum of the squares of three consecutive even numbers is always divisible by 4

[5 marks]

Question 10

AS-Level Examination Question from November 2021, Paper 1, Q10 (Edexcel)

A student is investigating the following statement about natural numbers, $\mathbb{N}$,

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.

[4 marks]

(b) Use a counterexample to show that the statement is not always true.

[1 mark]