Lesson 4

4.1 Non-Consecutive Numbers

Previously, in lesson 1, proofs about situations involving consecutive numbers were examined. However, suppose you wanted to prove a more general result such as the seemingly obvious fact that when any two odd numbers multiply, another odd number results. How do we drop the restriction that the two odd numbers be consecutive ?

4.2 The Non-Consecutive Odd Number Sequence

(i)   Complete the following table for the odd number sequence,

p	1	2	3	4	...	m	...	n	...
T_p	1	3			...		...		...

[2 marks]

(ii)   Hence prove that when any two odd numbers multiply, another odd number is the result.

[3 marks]

4.3 A Less Obvious use of Odd and Even

AS-Level Examination Question from June 2019, Paper 1, Q15, (Edexcel)

Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

This proof technique is known as "proof by exhaustion".

[4 marks]

4.4 Exercise

Any solution based entirely on graphical
or numerical methods is not acceptable

Marks Available : 40 marks

Question 1

Show that the sum of any two odd numbers is always an even number.

[2 marks]

Question 2

Show that 4 will always divide the product of any two even numbers.

[2 marks]

Question 3

For any natural number *n* use proof by exhaustion to show that $n^2 + 5$ is always either 1 or 2 more than a multiple of 4

[4 marks]

Question 4

A-Level Examination Question from June 2019, Paper 1, Q10 (i) (Edexcel)

Prove that for all $n \in \mathbb{N}$, $n^2 + 2$ is not divisible by 4

[4 marks]

Question 5

For any natural number n show that $n^2 + n + 2$ is not divisible by 3 by reasoning that n must be one of either $3k$ or $3k + 1$ or $3k + 2$ and then working through each of these possibilities in turn (a proof by exhaustion on divisibility by 3).

[4 marks]

Question 6

A-Level Examination Question from October 2020, Paper 2, Q16 (Edexcel)
Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

[4 marks]

Question 7

By making use of the result from question 6, prove that for any natural number, n,

$3n^2 - 1$ is never a perfect square.

[**4 marks**]

Question 8

(**i**) Explain why a perfect square that is even must be divisible by 4

[**3 marks**]

(**ii**) If x and y are odd numbers, prove that $x^2 + y^2$ can not be a perfect square.

[**6 marks**]

Question 9
* Challenge Question *

Prove that $n^3 + 5n$ is divisible by 6

[7 marks]