Lesson 6

6.1 Revision

Any solution based entirely on graphical
or numerical methods is not acceptable
Marks Available : 50 marks

Question 1

Marcus has a theory.

He believes that the product of an odd and an even number is never a perfect square.

Show that Marcus is wrong by finding a counterexample to disprove his theory.

[3 marks]

Question 2

Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

[5 marks]

Question 3

(**i**)   Prove that the difference of the squares of two consecutive even numbers is always divisible by 4.

[3 marks]

(**ii**)   Is the above statement true for odd numbers ?
Give a reason for your answer.

[3 marks]

Question 4

(**i**)   Work out, $3^2 + 4^2 + 5^2 + 6^2 + 7^2$

[**1 mark**]

(**ii**)   Prove that the sum of the squares of any five consecutive integers is divisible by 5

[**2 marks**]

(**iii**)   Prove that if five consecutive integers are squared, the mean of the squares exceeds the square of their median by 2

[**4 marks**]

(**iv**)   Hence, without using a calculator, write down the value of

$$\frac{98^2 + 99^2 + 100^2 + 101^2 + 102^2}{5}$$

[**2 marks**]

Question 5

AS-Level Examination Question from October 2020, Paper 1, Q13 (Edexcel)

(**a**)   Prove that for all positive values of *a* and *b*

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

[4 marks]

(**b**)   Prove, by counter example, that this is not true for all values of *a* and *b*.

[1 mark]

Question 6

Write down any 3-digit number *abc* then repeat the digits to form the 6-digit number

abcabc

Explain why the 6-digit number will be divisible by 7, by 11 and by 13

[5 marks]

Question 7

Divisibility by 7
Subtract twice the last digit from the number formed by the remaining digits.
Repeat as necessary.
If, at any stage, a result is obtained that is obviously divisible by 7 then the original number is also divisible by 7

(**i**)   Without using a calculator, show that 2331 is divisible by 7

[**2 marks**]

(**ii**)   Without using a calculator, show that 515011 is divisible by 7

[**2 marks**]

(**iii**)   Provide a proof that explains why the test for divisibility by 7 works

[**7 marks**]

Question 8

Examination Question from 2013, Q12 AH Maths (SQA)

Let *n* be a natural number.

For each of the following statements, decide whether it is true or false.

If true, give a proof; if false, give a counterexample.

(i)　　If *n* is a multiple of 9 then so is n^2

[3 marks]

(ii)　　If n^2 is a multiple of 9 then so is *n*

[3 marks]